WELFARE IMPACTS OF OPTIMAL VIRTUAL BIDDING IN A MULTI-SETTLEMENT ELECTRICITY MARKET WITH TRANSMISSION LINE CONGESTION

by

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

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With a goal of improving the performance of wholesale electricity markets, virtual financial products have been introduced. Virtual bids are purely financial instruments that may be used to speculate on the differences between prices in the forward and spot markets in a two-settlement electricity market. While there is evidence that price convergence across these markets can be induced by virtual bidding, other research has shown that the impacts on the welfare of market participants are less clear. The problem is that, while virtual bidding may narrow the price gap, if it does so by inflating the prices, then electricity consumers may not be better off.

Although some work has been done on the impacts of virtual transactions on welfare for the electricity market participants, that work provides an incomplete assessment because it ignores some important aspects of the electricity market system. In particular, the prior work due to Giraldo (2017) essentially ignores the electricity transmission network as well as the physical laws governing electricity flows. The objective of this research is to understand the effect of virtual transactions on electricity market efficiency (i.e. social welfare) using a model that explicitly includes the network as well as relationships that reflect the physical properties of electricity flows through a network (i.e. loop flow). The core research question is; what impact

does network congestion have on the welfare shifts caused by the participation of financial virtual traders?

This study employs models with multiple buses to analyze the welfare changes of electricity market participants in a network constrained multi-settlement electricity market. Integrating the network in the model enables a comparison of welfare changes between the simpler network-free models and a network-based model with the possibilities of line congestion and an explicit treatment of loop flow.

Using stylized two- and three-bus models, we estimated and compared the differences in welfare impact due to the introduction of virtual transactions between uncongested and congested networks as well as its heterogeneous impact on the different buses due to their location within the network. At the network level, congested lines amplify the welfare change due to introducing virtual transactions. We also found that the results from the simple models are broadly consistent in the complex network using the aggregate model of ISO-NE test case.

Results suggest that price convergence occurs with optimal virtual bidding in most cases, which is consistent with existing literature. The prices throughout the network in both forward and spot markets are changed, and there are welfare transfers among producers, consumers, and virtual traders relative to the market equilibrium without virtual bidding. Furthermore, the welfare impacts on market participants are not homogenous throughout the network. These implications should be considered in the design of regulations governing virtual transactions in the electricity market.

CHAPTER 1. INTRODUCTION

In the United States, Independent System Operators (ISO) and Regional Transmission Organizations (RTO), run the wholesale market that serve the majority of electricity consumers. The electricity markets administered by ISOs and RTOs commonly have a “two-settlement format” of forward and spot wholesale energy markets with an auction system for buying and selling electricity. The forward, or Day-Ahead (DA), market is executed on the day before the actual day when demand is served by dispatching electricity from generators. The DA market establishes financially binding commitments to inject and withdraw electricity from the electricity network. The spot, or Real-Time (RT) market, is a physical market executed on the day that electricity is dispatched to serve the load. System operators balance physical supply and demand for electricity in real time. An optimization model called the Optimal Power Flow model (OPF) determines market-clearing quantities in both the DA and RT markets. A byproduct of the solution to the OPF is spatially distinct marginal costs of satisfying demand at each node in the network. These marginal costs are constructed using the Lagrange multipliers from the OPF, and these marginal costs are called Locational Marginal Prices (LMPs). Quantities cleared in the DA market are valued at the associated LMPs. The RT market is treated as a “balancing market,” and deviations from the cleared quantities settled in the DA market are valued at the RT LMPs.

In the interest of improving the performance of wholesale electricity markets, virtual injection and withdrawal financial products have been introduced. Virtual transactions are financial products that may be used to speculate on differences between DA and RT prices in a two-settlement electricity market. Virtual traders do not control generation units or load-serving entities, although physical market participants can also use virtual bids either for speculative purposes or in an effort to provide a hedge against RT price volatility. Virtual traders can participate in wholesale markets without being obligated to have the capacity of physical generation or to serve electricity load. Virtual traders have a gross payoff or loss to a cleared, virtual bid in the DA market equal to the spread between DA and RT prices times the quantity cleared. Virtual bids are designed for traders to speculate or hedge on the price differences between DA and RT markets.

In this regard, the Pennsylvania-Jersey-Maryland Interconnection (PJM) market, the largest RTO in the world, has different types of virtual products including incremental offer, decremental bid, and up-to-congestion transactions. An incremental offer (INC) is a virtual supply offer. INCs are hourly and injection node-specific and consists of a price and energy quantity in MWs of generation to be injected into the network at the specified node. If an INC has a price that is less than the DA price at the injection node, then the bid will clear and possibly displace some physical generation commitment. In the RT market, the INC trader is obligated to purchase power to replace the generation they committed to supplying but do not. A decremental bid (DEC) is also hourly and withdrawal node-specific, and corresponds to a price and energy quantity in MWs of generation to be withdrawn from the network at the specified node. When the virtual bidder expects the DA price will be greater than the RT price, they may bid INC in order to make a profit from the price difference between DA and RT markets. INC may increase overall supply in the DA market and hence may decrease the DA prices. Up to congestion transaction (UTC) is virtual bid on the difference between prices from one node to another within the network, or in other words, they are virtual transactions focused on the value of the congestion price spread between two particular points. As with other virtual transactions, UTC is a bid in the DA market and are settled via the balancing RT market through offsetting transactions. Profitability occurs when the RT congestion price is greater than the purchased DA congestion price. However, UTC is different from other virtual bids because (a) they are directional – a bid on congestion from node A to node B is distinct from a bid on the congestion from node B to node A – and (b) UTC does not affect the commitment of generators in the day ahead market, but rather only their dispatch. The congestion price from node A to node B is defined as the nodal price at B minus the nodal price at A.

Despite the fact that virtual bids do not deliver or consume physical energy, some researchers believe that they can have an impact on both the forward and spot market prices. For example, when more generation units are committed in DA due to virtual, the more expensive subset of generation units in RT is available to be online when the realized demand in RT is higher than the forecasted demand in DA.

Opponents have claimed that the virtual transactions can make an expected profit without improving system performance such as unit commitment (Parsons et al. 2015). When the DA/RT price spread arises not from a deficiency of supply and demand, but from the electricity market design, such as the inconsistency in the granularity of time in determining settlement prices between the DA (hourly) and RT (every 5 minutes) market. Some have argued that virtual bids may provide unfavorable impacts on the electricity market because they can be used to manipulate the value of the other financial products such as Financial Transmission Rights (FTRs) while losing money on the virtual transactions (Ledgerwood and Pfeifenberger 2013; Celebi, Hajos, and Hanser 2010; Shan et al. 2016).

Proponents have argued that virtual bids increase market efficiency by eliciting price convergence (reducing the expected price differences) between DA and RT market prices (Borenstein et al. 2008; Hadsell 2011; Haugom and Ullrich 2012). For instance, Saravia (2003) studied the New York market and asserted that after the introduction of virtual bidding, the average price differences between DA and RT markets significantly decreased due to the increased level of competition and liquidity of virtual bids. Following the additional study of virtual products and price convergence in the NYISO by Hadsell and Shawky (2007), Bowden, Hu, and Payne (2009) found similar results in the MISO, and Jha and Wolak (2013), Li, Svoboda, and Oren (2015) and Woo et al. (2015) in the CAISO.

In the economics literature, efficiency is typically estimated by the size of the overall net economic surplus, also known as social welfare, resulting from resource use (Li and Tesfatsion 2011). General market efficiency analysis in the economic literature considers the magnitude of extracted efficient welfare from market participants. Social welfare is maximized by an appropriate resource distribution resulting from non-wasted resource usage. However, the literature on the welfare impact of virtual bids in electricity markets is thin.

The welfare analysis of the electricity market has been used to measure the impact of features of the electricity market such as demand response and line congestion. Walawalkar et al. (2008) estimate the welfare impact of demand response in the PJM market on market participants including producers and consumers. The study suggests that the major economic effect of demand response is welfare transfers from generators to the consumer having less price responsiveness. Willems and Küpper (2010) study the welfare change from price discriminating producers when transmission capacity is limited. They conclude that overall welfare is reduced as the congested lines bring profitable opportunities that mislead the production decisions of the monopolistic producer. Additionally, Mansur (2008) measures the welfare shifts due to the deregulation of the electricity market.

While there is clear evidence that price convergence can be induced by virtual bidding to address the touted benefits of virtual bids on market efficiency, other research has shown that the impacts on the welfare of market participants are less clear. The problem is that, while virtual bidding may narrow the price gap, if it does so by inflating the prices in both forward and spot markets, then electricity consumers may not be better off.

Giraldo et al. (2017) used welfare analysis to study the efficiency impact of a financial trader's virtual bid on wholesale electricity markets. The results of the study show that the virtual trading may transfer welfare between consumers and producers while financial traders have positive expected profits. The study employs a stylized parsimonious two-settlement market model based on a single node.

While (Giraldo et al. 2017) has been done on the impacts of virtual transactions on the welfare of the various electricity market participants, that work provides an incomplete assessment ignoring some important aspects of the electricity market system. In particular, important features of electricity networks that are eliminated in those models have to do with transmission lines and physical laws governing electricity flows. Previous studies lack theoretical models to measure the impact of virtual bids on market efficiency in the electricity network.

The simple models have limitations to applying the welfare results to the models of multiple buses in the network as the simplistic single bus model cannot accommodate the network congestion and loop flow. Due to the physical laws governing electricity networks, the single bus model cannot reflect these features precisely because there is no network. Using a model with more than a single bus demonstrates that introducing virtual transactions in a multi-bus network has somewhat different welfare impacts than the single bus case.

A transmission line becomes congested when energy delivered through the line reaches the thermal/physical limits of the lines and/or transformers. Transmission line congestion can necessitate changing the operations so as to avoid damaging the electricity network.

Consequently, the congested line may change the prices on the electricity network and thus impact the welfare of the market participants. When there is no congestion, LMPs will be identical throughout the network. If there is congestion, however, it can prevent full utilization of the lowest cost generation units resulting in a higher cost of satisfying demand.

The physical laws governing electricity flows in a multi-bus network, known as Kirchhoff's laws, are such that power flows cannot be directly controlled. This means that the existence of a path from an injection to a withdrawal node that has excess capacity does not necessarily mean that increasing the injection and withdrawal simultaneously can be accommodated by the network. This is because the existence of another path from the injection to the withdrawal node that is at maximum capacity would violate the line capacity constraint with the increase. This type of phenomenon cannot be observed in single or two-bus models. The result is that the physical laws governing electricity flows create distortions to the "ideal" operation of the network with generators dispatched in economic merit order, with attendant impacts on LMPs throughout the network. These distortions influence consumer and producer surplus, and thus have an impact on the welfare impacts due to the introduction of virtual transactions in the market.

The fundamental objective of this study is to understand how the properties of an electricity network (i.e. congestion and loop flow) affect the welfare impacts of introducing virtual transactions in an electricity market. Throughout the systematic examination of the simple model to multi-bus model, we expect to obtain the answers to the hypothesis following:

H_0^1: Having congestion in the network creates no differences in optimal bidding strategy of the financial trader and its welfare impact on market participants

H_0^2: Having optimal virtual bidding in the congested network creates homogeneous welfare impacts across the network

This study employs multi-bus networks with optimal power flow analysis to assess the welfare changes for electricity market participants in the network constrained multi-settlement electricity market. Introducing the network in the analysis enables this study to compare the results of welfare changes between the results from the simpler theoretical models and congested multi-bus models with loop flow.

We employ multiple types of the electricity networks and compare the welfare changes for market participants due to the introduction of virtual transactions. In this study, we analyze two- and three-bus networks using simulation to estimate the welfare impacts of the introduction of virtual transactions in a congested network. The two-bus network is the simplest network that includes transmission. The three-bus network is the simplest network that can incorporate a loop, and hence where Kirchhoff's Laws governing loop flows in an electrical network apply.

However, real electricity networks are far more complicated than these simple "sandbox" models, with more buses, transmission lines, and many possible patterns of congestion. The optimal bidding strategy in the more complicated network may involve multiple bids at alternative nodes, and hence, the overall welfare impact will likely be different. We use the 8 nodes ISO-NE Test System developed by Krishnamurthy, Li, and Tesfatsion (2016) as the basis for our analysis for a more complex electricity network

The market outcomes show that price convergence increases in expectation due to optimal virtual bidding in most cases, which is consistent with existing literature. The prices throughout the network in both forward and spot markets are affected, and there are welfare transfers among producers and consumers due to virtual bidding.

The results of the welfare analysis indicate that optimal virtual bidding tends to decrease consumer welfare when the expected spot price is equal to the cleared forward price, and congested lines in the network can magnify the welfare changes while virtual traders essentially always benefit. This is an important case because observations of market outcomes over time in PJM suggest that the forward prices appear to be an unbiased estimator of the spot price. Furthermore, the welfare impacts are not homogenous throughout the network. The heterogeneity depends on where an agent is located in the network relative to the congested line. That is, some generators may benefit while others lose, with the same being true for load-serving entities.

This thesis is structured as follows. In Chapter 2, we briefly present background on the wholesale electricity market. In Chapter 3, we address the method employed in the analysis and present algebraic welfare analysis in the single bus model. In Chapter 4, we present methods and formulations for welfare analysis in the multi-bus models. We analyze the results of the optimal bidding strategy and welfare impact with line capacity constraints using simple two- and three-bus networks in Chapter 5, and observe whether these impacts are consistent to the simple networks to a more complex network using ISO-NE test network in Chapter 6. In Chapter 7, we address the conclusions and discussions.

CHAPTER 2. WHOLESALE ELECTRICITY MARKET

The wholesale electricity market is composed of a Day-Ahead market (DA), which is a planning and financial forward market, and a Real-Time market (RT), which is a spot market. Participants in the electricity market are generators (suppliers), load-serving entities (consumers), or LSEs, and virtual bidders. Generators and load-serving entities are linked to physical resources at particular pricing nodes whereas a virtual bidder is not associated with physical generating capacity and does not have an obligation to serve load. A generator will only sell energy and load-serving entities will typically only buy energy at their respective nodes whereas virtual bidders may sell or buy electricity at any allowable node to bid in the network.

2.1 Multi Settlement Electricity Market

The multi-settlement market system is designed to facilitate the scheduling of generating resources to meet anticipated load. When the cleared purchases and sales of electricity settled at the DA price are not physically delivered in the RT market, they are bought or sold back at the RT price. When the realized electricity demand in the RT market is beyond the settled demand in the RT market additional generation is required to meet demand in the RT market. This additional generation is compensated at RT prices. Similarly, if demand is less than what cleared in the DA market, less generation is needed, and generation units pay for the differences at RT prices.

2.2 Locational Marginal Pricing

The wholesale electricity market is different from the other commodity markets due to the unstorable nature of electricity and physical laws governing electricity networks. Electricity cannot be practically stored at utility scale. Hence supply must always match the demand from second to second in real time. Following the physical laws of the electricity network, injected energy does not flow from the seller to the buyer of that energy on the shortest or most desirable path. The power flow follows the physical laws that are attributed to Kirchhoff and that are governed by the physical characteristics of the electricity network and the pattern of injections and withdrawals in the network.

A system operator controls the energy dispatch (injections into the network) to manage the power flow on the network within transmission constraints and security limits. The system operator needs to schedule generators before the actual dispatch moment because some generation units have start-up time to produce electricity. Unit commitment and economic dispatch are the major components of generation scheduling in the DA market. Unit commitment determines the unit start up and shut down schedules of generators in order to obtain least-cost dispatch. Economic dispatch is the determination of how to serve the loads distributed throughout the system at least cost.

Transmission line capacities create restrictions on the pattern of injections by generators and withdrawals by load-serving entities throughout the network in order to prevent damaging the transmission system. A congested transmission line limits the power flow between two physically connected nodes in the network, creating differences in the marginal value of power across the network. For this reason, wholesale electricity markets often use a spatial pricing mechanism called a locational marginal price. The LMP is the marginal cost of delivering energy at each node of the electricity network. These prices are defined by the solution to the economic dispatch problem.

Electricity buyers and sellers participate in the electricity market auction organized by market institutions that are variously called regional transmission organizations or independent systems operators. Generation units submit location specific willingness-to-sell offers with price-quantity pairs that are arranged into increasing step functions by the market institution. Load-serving entities (LSEs) submit location specific willingness-to-purchase bids with price-quantity pairs that are decreasing step functions, also by the node.

2.3 Optimal Power Flow

The Alternating Current Optimal Power Flow (ACOPF) model is an optimization problem that models how electricity flows across the electric grid following Kirchhoff's laws, and it is used to economically dispatch generation in real time in the least cost manner. The ACOPF optimization problem is, however, a tough problem to solve because it is a nonlinear, non-convex optimization problem with constraints that include trigonometric functions. The non-linearity that these equations significantly complicates the optimization problem (Hedman, Oren, and O'Neill 2011). Thus, it is common to use a linear approximation of the ACOPF problem.

The most common linear approximation of the ACOPF problem is known as the Direct Current Optimal Power Flow (DCOPF) problem. The DCOPF problem uses a simplification of the physical attributes of an electricity network that complicate ACOPF problem. The linearization is based on the assumption that resistance and line losses are insignificant, the per-unit voltage magnitude at each bus is 1 p.u.[1], and uses linear approximations to the sine and cosine functions about the point where the phase angle, θ, is equal to zero with cos(θ) = 1 and

[1] It is a convention to express voltage magnitude in per-unit (p.u.) terms that indicates the local value of specific bus relative to the nominal value. For example, when we set 100kV equals 1.00 p.u., the voltage magnitude at Bus 1 can be 1.02 p.u. that is 102kV.

$\sin(\theta) = \theta$ respectively. With these approximations, the DCOPF problem is a linear program, resulting in a problem that is much easier to solve than the nonlinear, non-convex ACOPF problem (L. Tang and Ferris 2015).

Formulations of the DCOPF problem often eliminate the phase angle variables and express the network constraints following Kirchhoff's laws regarding Power Transfer Distribution Factors, or PTDFs (Hedman, Oren, and O'Neill 2011). The PTDF formulation models flow on lines based on power injections and withdrawals at each bus (L. Tang and Ferris 2015).

The optimal power flow method solves the system operator's market welfare maximization problem using the Direct Current Optimal Power Flow (DCOPF) model with the physical constraints such as Kirchhoff's laws, line capacity constraints and generation-load energy balancing. Following the general practice of academia and industry, this study adopts Direct Current Optimal Power Flow (DCOPF) formulation for modeling the DA and RT markets.

2.4 Virtual Transactions

Over the last decade, virtual bidding has become a standard feature of wholesale energy markets in the U.S. Virtual bids are purely financial instruments and can represent a node-specific injection or withdrawal of electricity in the network in the DA market. Financial traders submitting virtual bids that are cleared in the DA market must resolve these contracts by buying or selling back the same number of megawatts of energy at the designated location and hour on the network in the RT market at the nodal RT price.

In PJM, virtual bids include increment offers (INCs), decrement bids (DECs) and up-to-congestion transactions (UTCs). INCs are submitted to the DA market to sell a given amount of energy at a specific node at a specified price. INCs can be considered as virtual supply as virtual generation offers in the DA Market. INCs are profitable when the DA clearing price exceeds the RT clearing price.

DECs are the exact opposite of INCs. DECs are submitted to the DA market to purchase energy at a specified node and price. DECs can be considered as virtual demand or virtual load bid in the DA market. DECs are profitable when the DA clearing price is lower than the RT clearing price.

UTCs are submitted to the DA market to hedge price differences between two points due to congestion. The UTCs designate a precise source (injection) and sink (withdrawal) nodes with a spread that specifies willingness-to-pay for the price differences between the source and the sink. The UTCs are cleared in DA market at the DA price of transmission (DA congestion price) that is the price difference between DA LMP at the sink and at the source when the reservation price of UTCs are larger than the DA price of transmission. The exact MW of UTCs that are cleared in DA market settle in the RT market at the RT congestion price.

The way to calculate profit is different based on bid direction. When the UTC bid specifies sink and source, it becomes prevalent direction if the sink LMP is greater than the source LMP. In this case, the DA congestion price becomes a cost and a RT congestion price becomes a revenue to calculate the expected profit: (UTCs in MW) x (RT congestion price– DA congestion price). If the sink LMP is less than the source LMP, it is counted as a counterflow direction. In this case, the DA congestion price becomes revenue, and RT congestion price becomes a cost to calculate the expected profit: UTCs x (DA congestion price– RT congestion price). It implies that when zero or miniscule DA congestion price is expected while RT congestion price can be at least higher than zero, UTC traders can reduce the risk by making a counter-flow bid with very low reservation cost that can be cleared by a very low DA congestion price.

UTCs and INCs/DECs have a distinct way to interact with the market. In PJM, they model INCs/DEC during the unit commitment process (Resource Scheduling Commitment in PJM) while they model UTCs during the economic dispatch process (Scheduling Pricing and Dispatch in PJM). In this sense, INCs/DECs can influence the unit commitment process while UTCs can affect the after unit commitment process, scheduling the dispatch of units that are already committed.

By nature, INCs/DECs are energy products while UTCs are transmission products. That means INCs/DECs correspond to energy costs of LMPs while UTCs correspond to congestion cost. When UTCs are cleared in DA market, they are counted as a pair of equal MW INCs and DECs at their source and sink nodes respectively. UTCs must be simultaneously cleared at both source and sink nodes with the same amount of MW while the independent INCs and DECs bid could be paired together, there is no certainty that they both would be cleared.

Virtual traders are motivated by speculation opportunities arising from expected discrepancies between prices in the DA and RT markets. The price divergence may occur due to unscheduled events such as unexpected demand changes, or generator or transmission outages. The two-settlement system settles all DA transactions at the DA prices and deviations from the DA cleared supply and demand at the RT prices. Thus, the speculation opportunities arise from the price differences between the DA and RT markets, providing potentially profitable opportunities for virtual traders.

If the realized RT price is higher than the DA price, then the trader that cleared a bid to demand power, which is virtual demand (DECs) has managed to “buy low and sell high”, making a profit on the transaction. The expected profit of the cleared virtual demand bid is the expected price difference (RT – DA) times the MWs of virtual demand, where the expectation is taken across the distribution of outcomes for the RT market.

As virtual bids compete with physical resources in the DA market, the speculating behavior of financial traders can affect how physical resources are committed and dispatched, and hence they influence market prices and system costs in DA and RT markets. For example, a virtual demand bid buying in DA in the hope of selling at a higher price in the RT market may result in the DA market clearing with more generation units committed and more generators dispatched to serve cleared demand bids including virtual demand bids.

The RT market outcomes can be changed by virtual bids (INCs and DECs, but not UTCs)[2] in the DA market because they can change the generation unit commitment plan in the DA market that is arranged to serve on the RT market. The altered unit commitment can have an impact on the actual dispatch in the RT market, thereby influencing RT prices.

2.5 Bilevel Programming

Bilevel programming is useful to model the sequential behavior of market participants when they influence each other's decisions. In this regard, the Bilevel programming can be of use to understand the behavior of virtual traders in the electricity market.

In the bilevel programming, two types of decision makers act as a hierarchical game. The leader optimizes an objective function subject to conditions imposed by optimal decisions of the follower. The leaders choices may impact the feasible region and the objective function of the follower's problem. The follower's reaction affects the leader's payoff and the scope of the

[2] INCs and DECs are considered the unit commitment process while UTCs are not.

leader’s initial selection. The leader's decision is perceived by the follower who reacts optimally regarding the observed action from the leader.

Some economic problems can be interpreted as bilevel programs. For instance, a Stackelberg leader-follower game can be viewed as a bilevel programming with the leader’s problem at the upper level and the follower’s problem at the lower level. The leader makes the first attempt to optimize the objective anticipating the responses of the follower. The follower selects the optimal strategy conditional on the leader’s behavior. The general structure is an instance of the bilevel programming setup.

Bilevel programming problems tend to be nonconvex optimization problems by nature. A common approach to investigate bilevel programming problems is to transform the bilevel optimization into the single-level optimization problem by replacing the lower level problem by its associated Karush-Kuhn-Tucker conditions. The Karush-Kuhn-Tucker (KKT) conditions are treated as a set of, typically nonlinear, constraints on the upper-level problem. The result is that the bilevel program is converted to a single level optimization problem, which is usually nonlinear and often nonconvex (Falk and Liu 1995).

Bilevel programming is appropriate for modeling optimization problems in which the top level optimizer (i.e. the one who moves first and anticipates the responses of the lower level agents) does not directly control a subset of the decision variables – namely, the ones chosen by the lower level agents. The problem of determining the profit-maximizing bids of market participants in a wholesale electricity market fits well in this context since the virtual market participants want to maximize their expected profit (or some index of the distribution of expected profit) while the system operator carries out a least-cost dispatch in order to minimize the overall system cost. In this context, the bilevel programming can be used to analyze electricity markets to reflect these conflicting goals (Hu and Ralph 2007; Fampa et al. 2007).

In the stylized electricity network model, the system operator's problem is the electricity market welfare maximization, which is the lower level programming problem. The two-settlement system of DA and RT markets can be viewed as optimization problems with recourse – decisions are made in the DA market, and decisions are subsequently made in the RT market that are conditional on the outcomes of random demand perturbations. The financial trader can be viewed as acting as the top-level agent, who places virtual bids with the motivation, assuming risk neutrality, of maximizing expected profit. At the lower level, the system operator maximizes electricity market welfare taking into account the bids of the financial trader.

CHAPTER 3. WELFARE ANALYSIS IN THE SINGLE BUS MODEL

We start with a simple stylized model with a single bus that permits mathematical analysis of the impact of INCs/DECs on market efficiency. We focus on a single hour and assume that the interactions between hours are negligible for the purposes of this stylized modeling exercise. Consumers are assumed to bid a fixed number of MW for the hour being modeled.

Giraldo et al. (2017) conducted welfare analysis to study the welfare impact of financial traders' virtual bids on wholesale electricity markets using constant elasticity of substitution electricity supply functions. The numerical results of the study show that, in the vast majority of case, the virtual transactions decrease electricity consumers' welfare and increase the producers' (generators') welfare while financial traders virtually always profit.

This study uses linear supply functions to develop a closed form solution for the welfare change of market participants when virtual trading is introduced in the market. The general results of optimal virtual bidding strategy and its welfare impacts on the market participants are consistent with the past literature. Certain extreme sets of parameters for DA and RT slope relative to the DA demand and the range of uncertainty, however, may change the qualitative welfare impact of market participants due to the different functional form used in this study.

3.1 Multi-Settlement Electricity Market with Virtual Transactions

Demand in the RT market, denoted by d2, is the sum of demand in the DA market, denoted by d1, plus a random deviation Δ.

$$\mathrm{d2} = \mathrm{d1} + \Delta \tag{1}$$

We assume that the financial trader knows the distribution of Δ.

In the real market, the supply function is a step function corresponding to price/quantity pairs bid by generators for the specific hour. The supply curve at any bus is an aggregate of individual step offer functions of generation units located at the specific bus. Here, we approximate this step function by a continuous, linear supply curve. Abstracting from the lumpiness of the step functions facilitates analysis of a stylized theoretical model. We assume that generators bid at their marginal cost without market power. ISO/RTOs have market monitoring practice to prevent the exercise of market power.

The linear supply function in DA market, denoted by S1, to meet the DA generation and DA demand, is characterized by the DA supply slope, denoted by b1, DA price, denoted by P1, and intercept a1.

$$\mathrm{S1} = \mathrm{g1} = \mathrm{d1} = \mathrm{a1} + \mathrm{b1} \cdot \mathrm{P1} \tag{2}$$

$$\begin{aligned} \mathrm{LMP1(d1)} &= \mathrm{P1(d1)} = -\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{d1}}{\mathrm{b1}} \\ &= \mathrm{P1(g1)} = -\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{g1}}{\mathrm{b1}} \end{aligned}$$

In a single bus model, the DA market price LMP1 is determined by the amount of DA generation, denoted by g1, in inverse DA supply function, denoted by P1 as in (2).

The RT supply curve is a piecewise linear function that is the same as the DA supply curve up to the DA cleared generation level g1. Beyond g1, the RT supply curve is again linear, but the slope, denoted by b2, is less steep than b1. The intercept of this portion of the curve, a2, is set so that the RT supply curve is continuous.

$$\begin{aligned} \mathrm{a2} &= \mathrm{d1} - \mathrm{b2} \cdot \mathrm{LMP1} = \mathrm{d1} - \mathrm{b2} \cdot \left(-\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{d1}}{\mathrm{b1}}\right) \\ &= \mathrm{g1} - \mathrm{b2} \cdot \mathrm{LMP1} = \mathrm{g1} - \mathrm{b2} \cdot \left(-\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{g1}}{\mathrm{b1}}\right) \end{aligned} \tag{3}$$

In this single node system, the generation in RT must equal the demand in RT which is cleared demand in DA plus the demand deviation in RT (g2=d2=d1+Δ).

The negative demand deviation in RT can be served by committed generation units in DA, and hence the LMPs for negative deviations are determined based on the DA supply curve. When generation in RT is greater than in DA, the RT market price, LMP2, is determined by the amount of RT generation, g2, in inverse RT supply function, P2 in (4).

$$\mathrm{LMP2} = \mathrm{P2}(\mathrm{d1}+\Delta) = \begin{cases} -\frac{\mathrm{a2}}{\mathrm{b2}} + \frac{\mathrm{d1}+\Delta}{\mathrm{b2}} = -\frac{\mathrm{d1}-\mathrm{b2}\cdot\left(-\frac{\mathrm{a1}}{\mathrm{b1}}+\frac{\mathrm{d1}}{\mathrm{b1}}\right)}{\mathrm{b2}} + \frac{\mathrm{d1}+\Delta}{\mathrm{b2}} & ,\Delta \geq 0 \\ -\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{d1}+\Delta}{\mathrm{b1}} & ,\Delta < 0 \end{cases} \quad (4)$$

$$= \mathrm{P2}(\mathrm{g2}) = \begin{cases} -\frac{\mathrm{a2}}{\mathrm{b2}} + \frac{\mathrm{g2}}{\mathrm{b2}} = -\frac{\mathrm{g1}-\mathrm{b2}\cdot\left(-\frac{\mathrm{a1}}{\mathrm{b1}}+\frac{\mathrm{g1}}{\mathrm{b1}}\right)}{\mathrm{b2}} + \frac{\mathrm{g2}}{\mathrm{b2}} & ,\mathrm{g2} \geq \mathrm{g1} \\ -\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{g2}}{\mathrm{b1}} & ,\mathrm{g2} < \mathrm{g1} \end{cases}$$

One thing to be noted is that the supply function in the RT balancing market is not identical to the one in the DA market due to the differences in the marginal costs of generation units used in the RT and DA markets.The slope of the DA supply curve is always greater than the slope of the RT supply curve (b1>b2).

In the DA market, the system operator has a wider availability of generation units to commit, and they are scheduled to dispatch in a least-cost manner to meet the forecasted demand. During the DA market clearing process, the system operator commits the units that will be available in the RT market. If realized demand is less than DA demand the committed generation units can be dispatched at a lower level than planned in the DA market.

In the RT market, when the realized demand deviates above the forecasted demand, the system operator has a limited set of the flexible generators to meet the increased demand. When there is a sudden increase in RT demand, only fast response resources meet the immediate positive demand deviation. The fast response resources, such as natural gas combustion turbine power plants, that can be brought online in a short time, generally have higher marginal costs. These generators usually have at least as high and usually higher marginal costs than what would have been available in the DA market. In this sense, the slope of abstracted inverse supply function in the RT balancing market is steeper than the one in the DA market (Isemonger 2006).

In the stylized model, all virtual bids are represented by the variable V. A positive V corresponds to an INC bid, and a negative V corresponds to a DEC bid. The virtual supply offer and demand bids influence the DA market price, LMP1, as if they bid supply and demand as in (5). While LMP1 represents a DA price without virtual, LMP1′ represents the DA price with virtual bidding. The ′ notation indicates the cases where virtual bids are used.

$$\begin{aligned} \mathrm{LMP1'(d1-V)} &= \mathrm{P1(d1-V)} = -\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{d1-V}}{\mathrm{b1}} \\ &= \mathrm{P1(g1')} = -\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{g1'}}{\mathrm{b1}} \end{aligned} \tag{5}$$

The supply curves are still continuous because the altered outcome of the DA market adjusts the a2 parameter as represented by (6), and the RT price with INC/DEC bids.

$$\begin{aligned} \mathrm{a2'} &= \mathrm{d1 - V - b2 \cdot LMP1'} = \mathrm{d1 - V - b2} \cdot \left(-\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{d1-V}}{\mathrm{b1}}\right) \\ &= \mathrm{g1' - b2 \cdot LMP1'} = \mathrm{g1' - b2} \cdot \left(-\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{g1'}}{\mathrm{b1}}\right) \end{aligned} \tag{6}$$

LMP2′, maintains continuity with a steeper slope as represented by (7).

$$\mathrm{LMP2'} = \mathrm{P2'(d1+\Delta)} = \begin{cases} -\frac{\mathrm{a2'}}{\mathrm{b2}} + \frac{\mathrm{d1+\Delta}}{\mathrm{b2}} = -\frac{\mathrm{(d1-V) - b2} \cdot \left(-\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{d1-V}}{\mathrm{b1}}\right)}{\mathrm{b2}} + \frac{\mathrm{d1+\Delta}}{\mathrm{b2}} & , \Delta \geq -\mathrm{V} \\ -\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{d1+\Delta}}{\mathrm{b1}} & , \Delta < -\mathrm{V} \end{cases} \tag{7}$$

$$= \mathrm{P2'(g2)} = \begin{cases} -\frac{\mathrm{a2'}}{\mathrm{b2}} + \frac{\mathrm{g2}}{\mathrm{b2}} = -\frac{\mathrm{g1' - b2} \cdot \left(-\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{g1'}}{\mathrm{b1}}\right)}{\mathrm{b2}} + \frac{\mathrm{g2}}{\mathrm{b2}} & , \mathrm{g2} \geq \mathrm{g1'} \\ -\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{g2}}{\mathrm{b1}} & , \mathrm{g2} < \mathrm{g1'} \end{cases}$$

3.2 Welfare of Market Participants: Formulations

This section provides equations for calculating expected economic surplus of market participants taking into account surplus in the DA market and distribution of adjustments to that surplus in the RT market. The consumer surplus calculation is the consumer payment in Day-Ahead (DA) plus balancing payments for the consumer in RT plus consumer value. The consumer value is calculated as d2 times reservation price (RP). The producer surplus calculation is equal to the payments to the generator in DA less the costs of generation minus balancing payments by the producer minus (plus) increases (decreases) in generation cost. The financial trader surplus calculation is payment to/by the financial trader in DA plus balancing payment by the financial trader in RT, depending on whether it is INC or DEC. Because the financial trader can always opt out, the expected surplus of the financial trader will always be positive, assuming risk neutrality. The expected change in social welfare due to the introduction of virtual trading is simply the difference in the market participants' welfare with and without virtual trading. The social welfare is a sum of consumer welfare and producer welfare. The virtual trader welfare is not counted in social welfare calculation since virtual bids are interrupter of the market rather than being a supplier or consumer of the electricity.

Consumer welfare without virtual trading

$$-\text{DA Payment by Consumer} + \text{RT Balance Payment to Consumer} + \text{Consumer Value}$$

$$= -\mathrm{d1} \cdot \mathrm{LMP1(d1)} + (\mathrm{d1} - \mathrm{d2}) \cdot \mathrm{LMP2(d2)} + \mathrm{d2} \cdot \mathrm{RP}$$

$$= -\mathrm{d1} \cdot \mathrm{LMP1(d1)} + \big(\mathrm{d1} - (\mathrm{d1} + \Delta)\big) \cdot \mathrm{LMP2}(\mathrm{d1} + \Delta) + \mathrm{d2} \cdot \mathrm{RP}$$

$$= -\mathrm{d1} \cdot \mathrm{LMP1(d1)} - \Delta \cdot \mathrm{LMP2}(\mathrm{d1} + \Delta) + \mathrm{d2} \cdot \mathrm{RP}$$

Consumer welfare with virtual trading

$$-\text{DA Payment by Consumer} + \text{RT Balance Payment to Consumer} + \text{Consumer Value}$$

$$= -\mathrm{d1} \cdot \mathrm{LMP1}'(\mathrm{d1} - \mathrm{V}) + (\mathrm{d1} - \mathrm{d2}) \cdot \mathrm{LMP2}'(\mathrm{d2}) + \mathrm{d2} \cdot \mathrm{RP}$$

$$= -\mathrm{d1} \cdot \mathrm{LMP1}'(\mathrm{d1} - \mathrm{V}) + \big(\mathrm{d1} - (\mathrm{d1} + \Delta)\big) \cdot \mathrm{LMP2}'(\mathrm{d1} + \Delta) + \mathrm{d2} \cdot \mathrm{RP}$$

$$= -\mathrm{d1} \cdot \mathrm{LMP1}'(\mathrm{d1} - \mathrm{V}) - \Delta \cdot \mathrm{LMP2}'(\mathrm{d1} + \Delta) + \mathrm{d2} \cdot \mathrm{RP}$$

ΔConsumer welfare due to the introduction of optimal virtual trading

$$\Delta \text{DA payment by consumer} + \Delta \text{ RT balance payment by the consumer}$$

$$= -\mathrm{d1} \cdot (\mathrm{LMP1}'(\mathrm{d1} - \mathrm{V}) - \mathrm{LMP1}(\mathrm{d1})) + (\mathrm{d1} - \mathrm{d2}) \cdot (\mathrm{LMP2}'(\mathrm{d2}) - \mathrm{LMP2}(\mathrm{d2}))$$

$$= -\mathrm{d1} \cdot (\mathrm{LMP1}'(\mathrm{d1} - \mathrm{V}) - \mathrm{LMP1}(\mathrm{d1})) - \Delta \cdot (\mathrm{LMP2}'(\mathrm{d2}) - \mathrm{LMP2}(\mathrm{d2}))$$

Producer welfare without virtual trading

$$\text{DA Payment to Producer} - \text{RT Balance Payment to Producer} - \text{Generation Cost}$$

$$= \mathrm{g1} \cdot \mathrm{LMP1}(\mathrm{d1}) - (\mathrm{g1} - \mathrm{g2}) \cdot \mathrm{LMP2}(\mathrm{d2})$$

$$-\left(\int_0^{\min(\mathrm{g1},\mathrm{g2})} \mathrm{P1}(\mathrm{g})\, \mathrm{dg} + \int_{\mathrm{g1}}^{\max(\mathrm{g1},\mathrm{g2})} \mathrm{P2}(\mathrm{g})\, \mathrm{dg}\right)$$

$$= \mathrm{d1} \cdot \mathrm{LMP1}(\mathrm{d1}) - \big(\mathrm{d1} - (\mathrm{d1} + \Delta)\big) \cdot \mathrm{LMP2}(\mathrm{d2})$$

$$-\left(\int_0^{\min(\mathrm{d1},\mathrm{d1}+\Delta)} \mathrm{P1}(\mathrm{g})\, \mathrm{dg} + \int_{\mathrm{d1}}^{\max(\mathrm{d1},\mathrm{d1}+\Delta)} \mathrm{P2}(\mathrm{g})\, \mathrm{dg}\right)$$

$$= \mathrm{d1} \cdot \mathrm{LMP1}(\mathrm{d1}) + \Delta \mathrm{LMP2}(\mathrm{d2})$$

$$-\left(\int_0^{\min(\mathrm{d1},\mathrm{d1}+\Delta)} \mathrm{P1}(\mathrm{g})\, \mathrm{dg} + \int_{\mathrm{d1}}^{\max(\mathrm{d1},\mathrm{d1}+\Delta)} \mathrm{P2}(\mathrm{g})\, \mathrm{dg}\right)$$

Producer welfare with virtual trading

$$\text{DA Payment to Producer} - \text{RT Balance Payment to Producer} - \text{Generation Cost}$$

$$= \mathrm{g1}' \cdot \mathrm{LMP1}'(\mathrm{d1} - \mathrm{V}) - (\mathrm{g1}' - \mathrm{g2}) \cdot \mathrm{LMP2}'(\mathrm{d2})$$

$$-\left(\int_0^{\min(\mathrm{g1}',\mathrm{g2})} \mathrm{P1}(\mathrm{g})\, \mathrm{dg} + \int_{\mathrm{g1}'}^{\max(\mathrm{g1}',\mathrm{g2})} \mathrm{P2}'(\mathrm{g})\, \mathrm{dg}\right)$$

$$= (\mathrm{d1} - \mathrm{V}) \cdot \mathrm{LMP1'}(\mathrm{d1} - \mathrm{V}) - \big((\mathrm{d1} - \mathrm{V}) - (\mathrm{d1} + \Delta)\big) \cdot \mathrm{LMP2'}(\mathrm{d2})$$

$$-\left(\int_0^{\min(\mathrm{d1-V,d1+\Delta})} \mathrm{P1(g)\,dg} + \int_{\mathrm{d1-V}}^{\max(\mathrm{d1-V,d1+\Delta})} \mathrm{P2'(g)\,dg}\right)$$

$$= \mathrm{d1} \cdot \mathrm{LMP1'}(\mathrm{d1} - \mathrm{V}) + (\Delta + \mathrm{V}) \cdot \mathrm{LMP2'}(\mathrm{d2})$$

$$-\left(\int_0^{\min(\mathrm{d1-V,d1+\Delta})} \mathrm{P1(g)\,dg} + \int_{\mathrm{d1-V}}^{\max(\mathrm{d1-V,d1+\Delta})} \mathrm{P2'(g)\,dg}\right)$$

ΔProducer welfare due to the introduction of optimal virtual trading

$$\Delta\ \text{DA payment to producer} + \Delta\ \text{RT balance payment to producer} +$$

$$\Delta\ \text{generation cost}$$

$$= (\mathrm{g1'} \cdot \mathrm{LMP1'}(\mathrm{d1} - \mathrm{V}) - \mathrm{g1} \cdot \mathrm{LMP1}(\mathrm{d1})) + \{(\mathrm{g2} - \mathrm{g1'}) \cdot \mathrm{LMP2'}(\mathrm{d2}) - (\mathrm{g2} - \mathrm{g1}) \cdot$$

$$\mathrm{LMP2}(\mathrm{d2})\} + \left\{-\left(\int_0^{\min(\mathrm{g1',g2})} \mathrm{P1(g)\,dg} + \int_{\mathrm{g1}}^{\max(\mathrm{g1',g2})} \mathrm{P2'(g)\,dg}\right) +\right.$$

$$\left.\left(\int_0^{\min(\mathrm{g1,g2})} \mathrm{P1(g)\,dg} + \int_{\mathrm{g1}}^{\max(\mathrm{g1,g2})} \mathrm{P2(g)\,dg}\right)\right\}$$

$$= ((\mathrm{d1} - \mathrm{V}) \cdot \mathrm{LMP1'}(\mathrm{d1} - \mathrm{V}) - \mathrm{d1} \cdot \mathrm{LMP1}(\mathrm{d1})) + \big\{\big((\mathrm{d1} + \Delta) - (\mathrm{d1} - \mathrm{V})\big) \cdot$$

$$\mathrm{LMP2'}(\mathrm{d2}) - \big((\mathrm{d1} + \Delta) - \mathrm{d1}\big) \cdot \mathrm{LMP2}(\mathrm{d2})\big\} + \left\{-\left(\int_0^{\min(d1-V,\mathrm{d2})} \mathrm{P1(g)\,dg} +\right.\right.$$

$$\left.\left.\int_{d1-V}^{\max(d1-V,\mathrm{d2})} \mathrm{P2'(g)\,dg}\right) + \left(\int_0^{\min(d1,\mathrm{d2})} \mathrm{P1(g)\,dg} + \int_{\mathrm{d1}}^{\max(\mathrm{d1,d2})} \mathrm{P2(g)\,dg}\right)\right\}$$

$$= ((\mathrm{d1} - \mathrm{V}) \cdot \mathrm{LMP1'}(\mathrm{d1} - \mathrm{V}) - \mathrm{d1} \cdot \mathrm{LMP1}(\mathrm{d1})) + \{(\Delta - \mathrm{V}) \cdot \mathrm{LMP2'}(\mathrm{d2}) - \Delta \cdot$$

$$\mathrm{LMP2}(\mathrm{d2})\} + \left\{-\left(\int_0^{\min(d1-V,\mathrm{d2})} \mathrm{P1(g)\,dg} + \int_{d1-V}^{\max(d1-V,\mathrm{d2})} \mathrm{P2'(g)\,dg}\right) +\right.$$

$$\left.\left(\int_0^{\min(d1,\mathrm{d2})} \mathrm{P1(g)\,dg} + \int_{\mathrm{d1}}^{\max(\mathrm{d1,d2})} \mathrm{P2(g)\,dg}\right)\right\}$$

Financial Trader welfare

DA Payment to Financial Trader - RT Payment by Financial trader

$$= \mathrm{V} \cdot \mathrm{LMP1'}(\mathrm{d1} - \mathrm{V}) - \mathrm{V} \cdot \mathrm{LMP2'}(\mathrm{d2})$$

$$= \mathrm{V} \cdot (\mathrm{LMP1'}(\mathrm{d1} - \mathrm{V}) - \mathrm{LMP2'}(\mathrm{d2}))$$

3.3 Price Change by Optimal Virtual Bid with Zero Expected Demand Deviation

Giraldo (2017) shows that the optimal virtual bid, when the expectation of demand deviations between the day ahead (DA) and real-time (RT) markets is zero, is a decrement (DEC) bid. This has the effect of shifting the equilibrium quantity in the DA market to the right and shifts the equilibrium DA price up relative to the case of no virtual bidding. This shifts the point at which the RT supply becomes steeper to the right as is illustrated in figure 3.1. Thus, with a zero expected demand deviation, the RT supply with virtual bidding will lie above (at quantities less than or equal to the DA demand without virtual bids) or below (above the DA demand without virtual bids).

Because the support and probability distribution of the RT demand is unchanged and price as a function of RT demand is on or below the price function without virtual bids, the expected RT price will be lower in the presence of virtual bids. Thus, price convergence always occurs when the optimal bidding strategy id DEC and the expected RT demand deviation is zero – DA price increases, and expected RT price decreases.

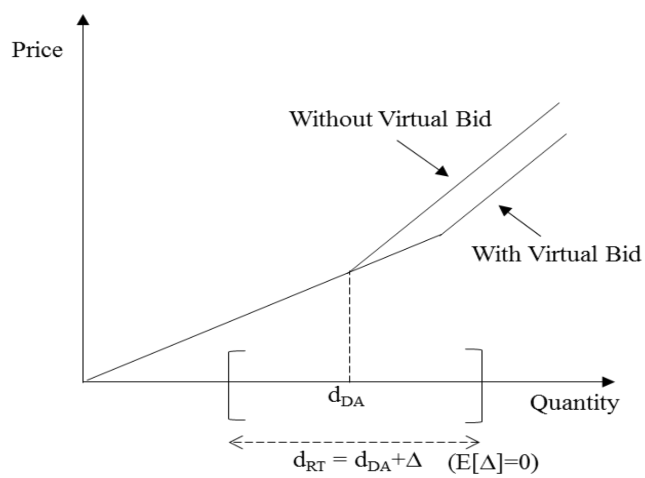

Figure 3.1 RT Supply (Price as a Function of Demand) with and without a Virtual DEC Bid

3.4 Welfare of Market Participants: Graphical Representations

A graphical representation helps illustrate the welfare calculation. We will demonstrate the three cases of RT demand realization relative to DA demand: 1) RT demand is identical to DA demand, 2) RT demand is less than DA demand, 3) RT demand is greater than DA demand.

Chapter 3.4.1 shows all three cases without virtual bids. Chapter 3.4.2 shows the deterministic case when RT demand is identical to DA demand with different bidding strategies. Chapter 3.4.3 shows the stochastic case when RT demand is less than DA demand different bidding strategies and Chapter 3.4.4 shows the stochastic case when RT demand is greater than DA demand with different bidding strategies. Some cases are not profitable bidding strategy, however, we will investigate them as well for the sake of completeness.

3.4.1 Graphical Representations: Without Virtual Bids

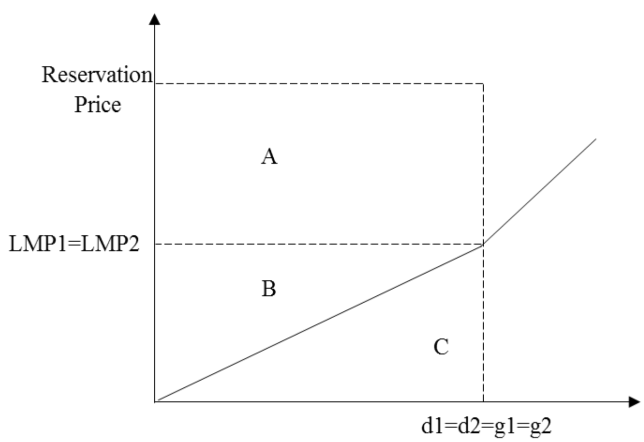

Figure 3.2 Welfare calculation when RT demand is the same as DA demand

Figure 3.2 displays the case where there is no demand deviation and no financial trader. Identical demands in Day-Ahead (DA) and Real-Time (RT) markets make DA generation (g1) and RT generation (g2) identical. The prices are the same as they are set along with the equal supply curve in DA and RT market.

For consumer surplus, DA payment by the consumer is d1·LMP1, the area of B+C, and consumer value is d2·RP, the area of A+B+C. The total consumer surplus is the area of A, or –(B+C)+(A+B+C). For producer surplus, DA payment to the producer is g1·LMP1, the area of B+C, and the generation cost is the area under the inverse supply curve up to d2, C. Thus, the total producer surplus is the area of B, or (B+C)-(C). The calculations have no significant dissimilarity from standard economic welfare textbooks.

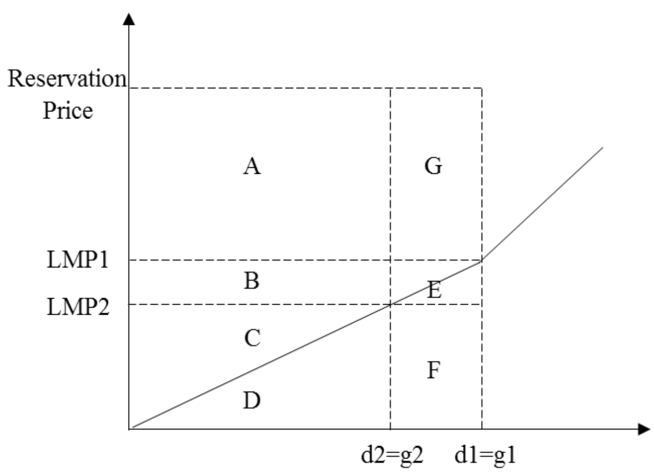

Figure 3.3 Welfare calculation when RT demand is less than DA demand

Figure 3.3 shows the case when cleared demand in DA is larger than realized demand in RT. Determined generation in DA must be equivalent to the cleared demand in DA and dispatched generation in RT must be equal to the realized demand in RT in this single bus environment.

For consumer surplus, the DA payment by the consumer is d1·LMP1, the area of B+E+C+D+F, RT balance payment to the consumer is (d1-d2)·LMP2, the area of F and consumer value is d2·(Reservation Price), the area of A+B+C+D. The total consumer surplus is the area of A-E, or –(B+E+C+D+F)+(F)+(A+B+C+D).

For producer surplus, DA payment to the producer is g1·LMP1, the area of B+C+D+E+F, the RT balancing payment to the producer is -(g1-g2)·LMP2, the area of F and the generation cost is the area under the inverse supply curve up to d2, D. The total producer surplus is the area of B+C+E, or (B+C+D+E+F)-(F)-(D).

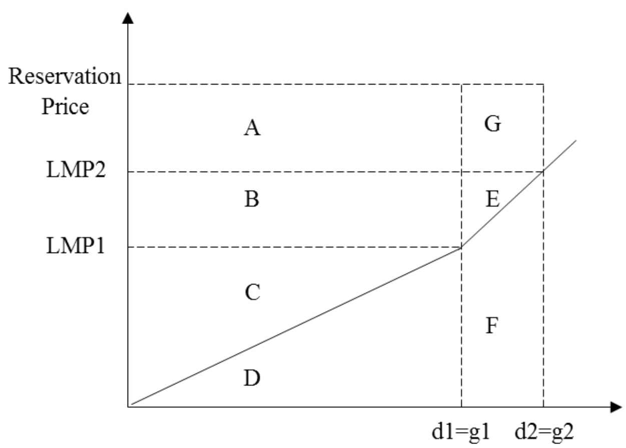

Figure 3.4 Welfare calculation when RT demand is greater than DA demand

Figure 3.4 shows the case when cleared demand in DA is lower than realized demand in RT. For consumer surplus, the DA payment by the consumer is $\mathrm{d1} \cdot \mathrm{LMP1}$, the area of C+D, RT balance payment to the consumer is $(\mathrm{d1} - \mathrm{d2}) \cdot \mathrm{LMP2}$, the area of E+F and consumer value is $\mathrm{d2} \cdot \mathrm{RP}$, the area of A+B+C+D+E+F+G. The total consumer surplus is the area of A+B+G, -(C+D)-(E+F)+A+B+C+D+E+F+G.

For producer surplus, the DA payment to the producer is g1 · LMP1, the area of C+D, RT balancing payment to the producer is $-(g1 - g2)$ · LMP2, the area of E+F and the generation cost is the area under the inverse supply curve up to d2, D+F. The total producer surplus is the area of C+E, or C+D+E+F–(D+F).

Supply and demand elasticity differences and balancing payment rules are the sources of surplus transfer between the case of the single market and the case of the two-settlement market with demand deviation.

3.4.2 Graphical Representations: With Virtual Bids, RT demand is identical to DA demand

The represented cases above have no virtual transactions. Virtual transactions in the two-settlement electricity market influence the cleared generation and demand bids in the DA market and hence influence the prices in DA and RT markets. The changes in the volume of cleared bids and prices alter the welfare of market participants, including consumer, producer, and financial trader.

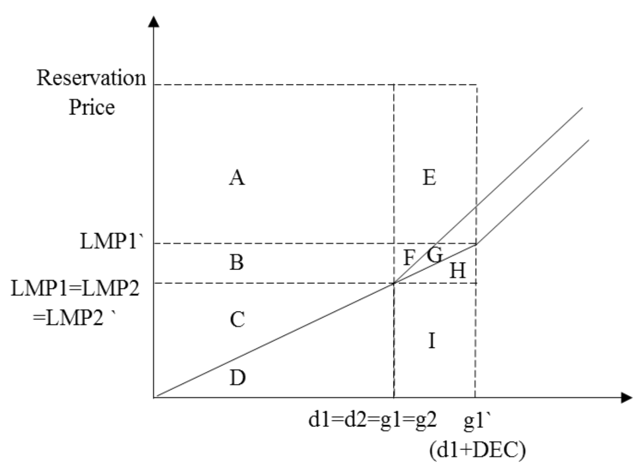

Figure 3.5 Welfare calculation when RT demand is identical to DA demand with a DEC

Figure 3.5 is the case where DA demand is identical to RT demand with a virtual DEC bid. Without the financial trader, the electricity market settles as single spot market where d1=d2=g1=g2. Hence the consumer surplus is A+B, and the producer surplus is C. With the financial trader, the consumer surplus is A, the producer surplus is B+C+F+G+H and financial trader's loss is F+G+H. When DA demand and RT demand are identical with a virtual DEC bid, consumer surplus decreases, producer surplus increases and financial trader surplus becomes negative. In this type of deterministic case, the virtual trader would not participate in the real world. However, we cover this case for the sake of completeness.

Calculating economic surplus without DECs, the DA payment by the consumer is d1·LMP1 or the area of C+D. There is no RT balancing payment by the consumer because d2 is equal to d1, and consumer value is d2·RP, is the area of A+B+C+D. The total consumer surplus is the area of A+B, or -(C+D)+(A+B+C+D). The DA payment to the producer is g1·LMP1 or the area of C+D. There is again no RT balance payment to the producer, since g2 is equal to g1, and the generation cost is the area under the inverse supply curve up to d2, or D. The total producer surplus is the area of C, or (C+D)-(D).

Now consider the case where virtual transactions are introduced. In this case, the changes in generation and price due to DECs have an impact on the welfare calculations. The DA payment by the consumer is d1·LMP1´, the area of B+C+D. There are no RT balancing payments by the consumer because RT demand, d2, is equal to DA demand, d1, and consumer value is d2·RP, the area of A+B+C+D. The total consumer surplus is the area of A, or -(B+C+D)+(A+B+C+D). DA payments to the producer is g1´·LMP1´, the area of B+C+D+F+G+H+I, RT balance payment to the producer is -(g1´-g2)·LMP2, the area of H, which is negative when g2 is less than g1´, and the generation cost is the area under the inverse

RT supply curve up to d2, D. The total producer surplus is the area of B+C+F+G+H, or (B+C+D+F+G+H+I)-(I)-(D).

The financial trader surplus is –(F+G). DA payment to the financial trader is V·LMP1´, the area of F+G+H. RT payment by the financial trader is INCs·LMP2, the area of H. DECs are settled as virtual demand bids. The total financial trader surplus is the area of –(F+G+H), or -(F+G+H+I)+(I) which is negative. Consequently, the strategy of bidding a DEC would not be favored by financial trader expecting RT demand is identical to DA demand.

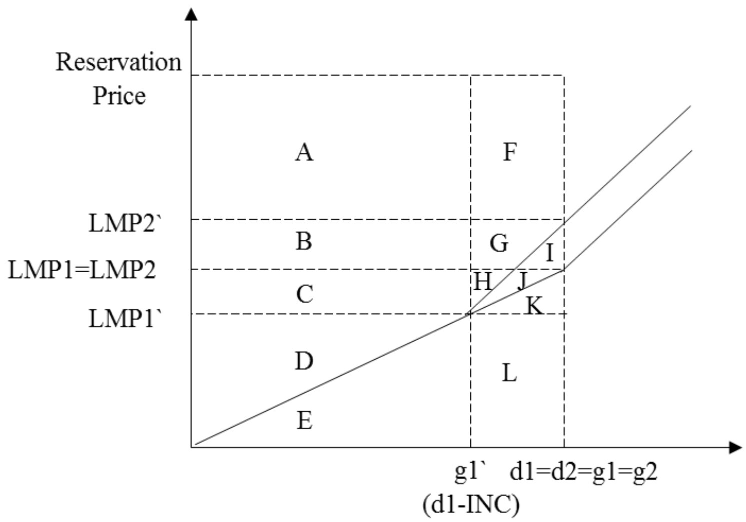

Figure 3.6 Welfare calculation when RT demand is identical to DA demand with an INC

Figure 3.6 is the case when DA demand is identical to RT demand with INC. Without the financial trader, the electricity market settles as a single spot market where d1=d2=g1=g2. The consumer surplus is A+B+F+G+I, and the producer surplus is C+D+H+J. With the financial trader, the consumer surplus is A+B+C+F+G+H+I+J+K, the producer surplus is D+G+H and financial trader loss is G+H+I+J+K. When DA demand and RT demand are identical and the virtual trader bids INCs, consumer surplus increases, producer surplus decreases, and financial trader surplus becomes negative. Again, we cover this case for the sake of completeness.

Calculating economic surplus without INCs, the DA payment by the consumer is d1·LMP1, the area of C+D+E+H+J+K+L, and there is no RT balancing payment by the consumer since d2 is equal to d1. Total consumer value is d2·RP, the area of A+B+C+D+E+F+G+H+I+J+K+L. So total consumer surplus is the area of A+B+F+G+I, or -(C+D+E+H+J+K+L)+(A+B+C+D+E+F+G+H+I+J+K+L).

The DA payment to the producer is g1·LMP1, the area of C+D+E+H+J+K+L, and there is no RT balancing payment to the producer, since g2 is equal to g1, and the generation cost is the area under the inverse supply curve up to d2, E+K+L. The total producer surplus is the area of C+D+H+J, (C+D+E+H+J+K+L)-(E+K+L).

The introduction of INC bids changes a generation and price in the DA and RT markets. The DA payment by the consumer is d1·LMP1´, the area of D+E+L There are no RT balancing payments by the consumer is (d1-d2)·LMP2´ because the d2 is equal to d1, and consumer value is d2·RP, the area of A+B+C+D+E+F+G+H+I+J+K+L. The total consumer surplus is the area of A+B+C+F+G+H+I+J+K, or -(D+E+L)+(A+B+C+D+E+F+G+H+I+J+K+L).

The DA payment to the producer is g1´·LMP1´, the area of D+E. The RT balancing payment to the producer is -(g1´-g2)·LMP2´, the area of G+H+I+J+K+L, which is positive when g2 is larger than g1´, and the generation cost is the area under the inverse supply curve up to d2, E+I+J+K+L. The total producer surplus is the area of D+G+H, or (D+E)+(G+H+I+J+K+L)-(E+I+J+K+L).

The financial trader surplus is -(H+J), which is negative in this case. DA payment to the financial trader is V·LMP1´, the area of L. RT payment by the financial trader is V·LMP2´, the area of G+H+I+J+K+L. The total financial trader surplus is the area of –(G+H+I+J+K), or (L)-(G+H+I+J+K+L) which is negative. Consequently, bidding INCs would not be favored by the financial trader when RT demand is identical to DA demand. Thus, the optimal bidding strategy for a financial trader with unbiased deviation is to avoid bidding. Therefore, with zero expected demand deviation, the presence of uncertainty of realized demand is essential to make profitable virtual bidding.

3.4.3 Graphical Representations: With Virtual Bids, RT demand is less than DA demand

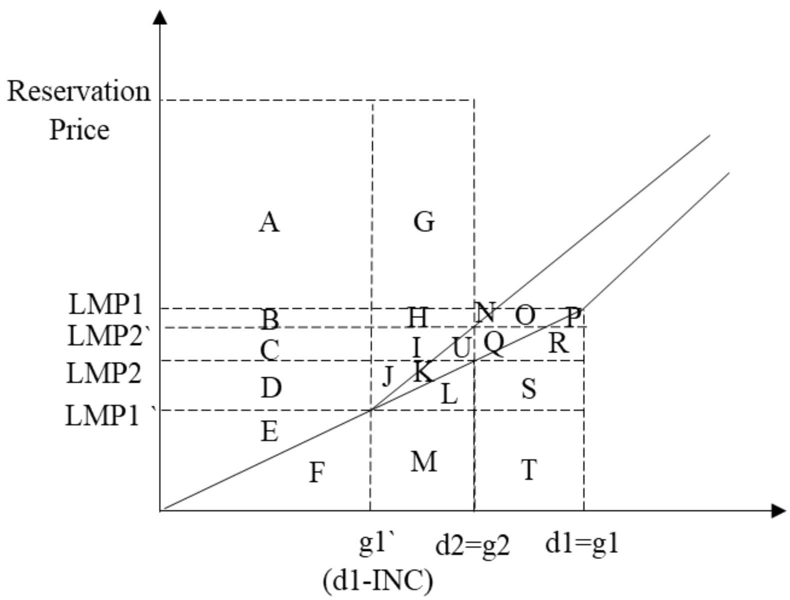

Figure 3.7 Welfare calculation when RT demand is less than DA demand with an INC

Figure 3.7 is the case when DA demand is larger than RT demand with INCs making the committed generation in DA less than realized demand in RT. Without the financial trader, the consumer surplus is A+G-(N+O+P+Q+R), and the producer surplus is B+C+D+E+G+H+I+J+K+N+O+P+Q+R+U. With the financial trader, the consumer surplus is A+B+C+D+G+H+I+J+K+L+Q+R+S, the producer surplus is F+J-U and financial trader surplus is –(I+U+J+K+L+Q+R+S). When DA demand is larger than RT demand with INCs setting committed DA generation less than RT demand, consumer surplus increases, producer surplus decreases, and financial trader surplus is negative.

Calculating economic surplus without INCs, the DA payment by the consumer is d1·LMP1, the area of B+C+D+E+F+H+I+J+K+L+M+N+O+P+Q+R+S+T+U. The RT balancing payment by the consumer is (d1-d2)·LMP2, the area of S+T, which is positive when d2 is less than d1, and consumer value is d2·RP, the area of A+B+C+D+E+F+G+H+I+J+K+L+M+U. The total consumer surplus is the area of A+G-(N+O+P+Q+R),

-(B+C+D+E+F+H+I+J+K+L+M+N+O+P+Q+R+S+T+U)

+(S+T)+(A+B+C+D+E+F+G+H+I+J+K+L+U).

The DA payment to the producer is g1·LMP1, the area of B+C+D+E+F+H+I+J+K+L+M+N+O+P+Q+R+S+T+U, RT balance payment to the producer is -(g1-g2)·LMP2, -(S+T), which is negative when g2 is less than g1, and the generation cost is the area under the inverse supply curve up to d2, F+L+M. The total producer surplus is the area of B+C+D+E+G+H+I+J+K+N+O+P+Q+R+U, or

(B+C+D+E+F+H+I+J+K+L+M+N+O+P+Q+R+S+T+U)-(S+T)-(F+L+M).

With INCs, the cleared generation in DA (g1´) must meet the overall (real plus virtual) demand in the DA market, the physical demand bid minus the virtual bid V (g1´=d1-INC). That makes the overall demand less than physical demand bids in DA requiring less generation relative to without INCs, virtual supply. Cleared generation in DA with INCs (g1´) set the newly cleared DA market price (LMP1´), lower than cleared DA market price without INCs (LMP1).

Based on the changed DA generation and price by INCs, the DA payment by the consumer is $d1 \cdot LMP1'$, the area of E+F+M+T, and the RT balancing payment to the consumer is $(d1 - d2) \cdot LMP2'$, the area of Q+R+S+T, which is positive when d2 is less than d1, and the consumer value is $d2 \cdot RP$ or the area of A+B+C+D+E+F+G+H+I+J+K+L+M. The total consumer surplus is the area of A+B+C+D+G+H+I+J+K+L+Q+R+S+U, or –(E+F+M+T)+(Q+R+S+T)+ (A+B+C+D+E+F+G+H+I+J+K+L+M+U).

The DA payment to producer is $g1' \cdot LMP1'$, is the area of E+F, and the RT balancing payment to the producer is $-(g1' - g2) \cdot LMP2'$, or the area of J+K+L+M, which is positive when g2 is greater than g1´, and the generation cost is the area under the RT inverse supply curve up to d2, F+U+K+L+M. Thus, total producer surplus is the area of E+J-U, or (E+F)+(J+K+L+M)-(F+U+K+L+M).

The financial trader surplus (loss) is –(I+J+K+O), which is negative in this case. DA payment to the financial trader is V·LMP1´, the area of M+T. The RT payment by the financial trader is V·LMP2´, or the area of I+U+J+K+L+M+Q+R+S+T. The supply offers including the financial trader's virtual supply offer need to be paid in DA market and need to pay balance payment in RT market. The total financial trader surplus is the area of –(I+U+J+K+L+Q+R+S) which is negative. Consequently, bidding an INC that results in committed generation less than RT generation when RT demand is less than DA demand is not leveraged for financial trader expecting RT demand is less than DA demand.

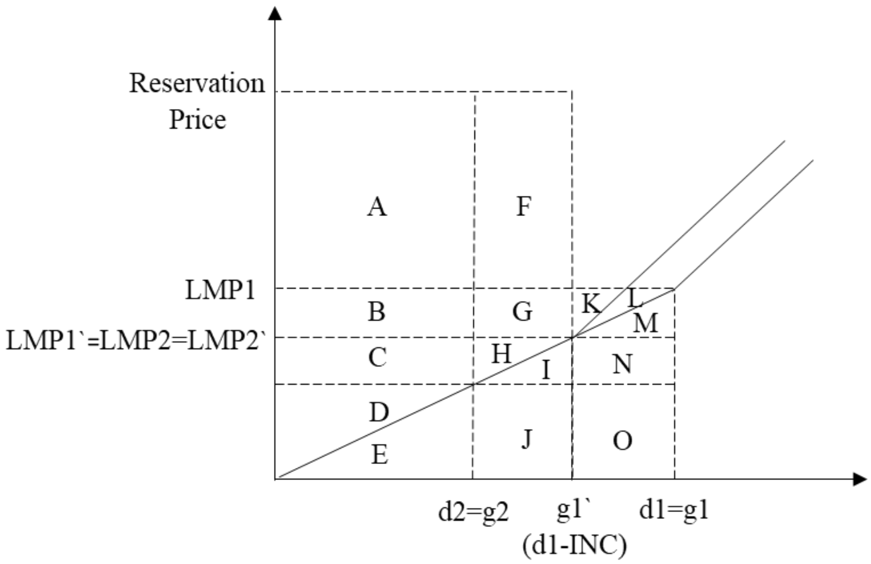

Figure 3.8 Welfare calculation when RT demand is less than DA demand with an INC

Figure 3.8 displays the case when DA demand is larger than RT demand with an INC bid by the financial trader, making the committed generation in DA greater than realized demand in RT and less than cleared demand in DA. Without the financial trader, consumer surplus is A-(G+H+I+K+L+M), and the producer surplus is B+C+D+G+H+I+K+L+M+N. With the financial trader, the consumer surplus is A+B-(H+I+N), the producer surplus is C+D+H+I and financial trader surplus is N. When DA demand is larger than RT demand with INCs, consumer surplus increases, producer surplus decreases, and financial trader surplus is positive.

Calculating the consumer surplus without INCs, the DA payment by the consumer is d1·LMP1 or the area of B+C+D+E+G+H+I+J+K+L+M+N+O and the RT balancing payment by the consumer is (d1-d2)·LMP2 or the area of J+O when d2 is less than d1. Since the consumer value is d2·RP, the area of A+B+C+D+E, the total consumer surplus is the area of A-(G+H+I+K+L+M), or -(B+C+D+E+G+H+I+J+K+L+M+N+O)+(J+O)+(A+B+C+D+E).

The DA payment to the producer is g1·LMP1 or the area of B+C+D+E+F+G+H+I+J+K+L+M+N+O and the RT balancing payment to the producer is -(g1-g2)·LMP2, or –(J+O) when g2 is less than g1. The generation cost is the area under RT inverse supply curve up to d2, or E. The total producer surplus is the area of B+C+D+G+H+I+K+L+M+N, or (B+C+D+E+F+G+H+I+J+K+L+M+N+O)-(J+O)-E.

DA generation and clearing price are changed by an INC bid, and the DA payment by the consumer is $d1 \cdot LMP1'$, or the area of C+D+E+ H+I+J+N+O. The RT balancing payment to the consumer is $(d1 - d2) \cdot LMP2$, or the area of J+O, when d2 is less than d1. The consumer value is $d2 \cdot RP$ or the area of A+B+C+D+E. Thus, the total consumer surplus is the area of A+B-(H+I+N), or –(C+D+E+ H+I+J+N+O)+(J+O)+(A+B+C+D+E).

The DA payment to the producer is $g1' \cdot LMP1'$ or the area of C+D+E+H+I+J, and the RT balancing payment to the producer is $-(g1' - g2) \cdot LMP2$, or the area of –J when g2 is less than g1´. The generation cost is the area under the inverse supply curve up to d2, or E. Thus, the total producer surplus is the area of C+D+H+I, or (C+D+E+H+I+J)-(J)-(E).

The financial trader surplus is N, which is positive in this case. The DA payment to the financial trader is V·LMP1´, or the area of N+O. RT payment by the financial trader is V·LMP2 or the area of –O. The supply offers including the financial trader's virtual supply offer need to be paid in DA market and need to pay balance payment in RT market as if supplying energy. The total financial trader surplus is the area of N, (N+O)-O. Bidding an INC makes the committed generation in DA greater than realized demand in RT and less than cleared demand in DA would generate positive returns to the financial trader.

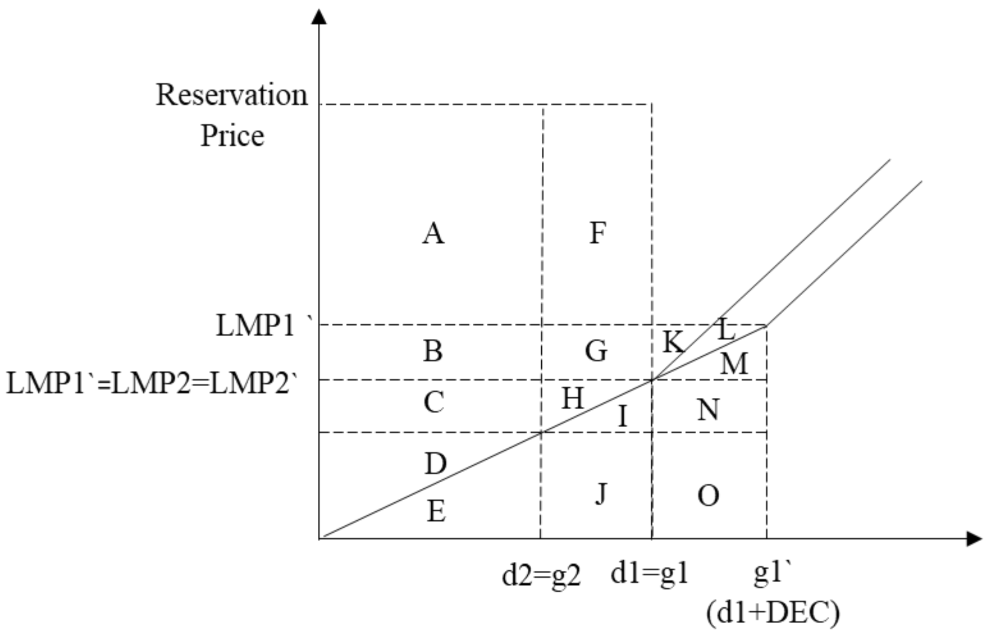

Figure 3.9 Welfare calculation when RT demand is less than DA demand with a DEC

Figure 3.9 is the case when DA demand is larger than RT demand with a DEC offer. Without the financial trader, consumer surplus is A+B-(H+I), and the producer surplus is C+D+H+I. With the financial trader, the consumer surplus is A-(G+H+I), the producer surplus is B+C+D+G+H+I+K+L+M+N and financial trader surplus is -(K+L+M+N).

Calculating economic surplus without DECs, DA payment by the consumer is d1·LMP1, the area of C+D+E+H+I+J, RT balance payment by the consumer is (d1-d2)·LMP2, the area of J, which is positive when d2 is less than d1, and consumer value is d2·RP, the area of A+B+C+D+E. The total consumer surplus is the area of A+B-(H+I), or –(C+D+E+H+I+J)+(J)+(A+B+C+D+E).

The DA payment to the producer is g1·LMP1, the area of C+D+E+H+I+J and the RT balancing payment to the producer is -(g1-g2)·LMP2, the area of –J when g2 is less than g1. The generation cost is the area under the inverse supply curve up to d2, or E. The total producer surplus is the area of C+D+H+I, or (C+D+E+H+I+J)-(J)-(E).

Offering a DEC makes the overall demand larger than physical bids, so that requires more generation relative to the case without DECs. Cleared generation in DA with a DEC bid (g1′) sets the new DA market price (LMP1′), which is higher than DA market price without DEC bid (LMP1).

Based on the changed DA generation and price due to the DECs, the DA payment by the consumer is $\mathrm{d1} \cdot \mathrm{LMP1}'$, or the area of B+C+D+E+G+H+I+J, and the RT balancing payment to the consumer is $(\mathrm{d1} - \mathrm{d2}) \cdot \mathrm{LMP2}$, or the area J when d2 is less than d1. The consumer value is $\mathrm{d2} \cdot \mathrm{RP}$ or the area of A+B+C+D+E. Thus the total consumer surplus is the area of A-(G+H+I), or –(B+C+D+E+G+H+I+J)+(J)+(A+B+C+D+E).

DA payment to producer is $\mathrm{g1}' \cdot \mathrm{LMP1}'$ or the area of B+C+D+E+G+H+I+J+K+L+M+N+O, and the RT balancing payment to the producer is $-(\mathrm{g1}' - \mathrm{g2}) \cdot \mathrm{LMP2}$, the area of –(J+O) when g2 is less than g1′, and the generation cost is the area under the inverse supply curve up to d2, or E. The total producer surplus is the area of B+C+D+G+H+I+K+L+M+N, or (B+C+D+E+G+H+I+J+K+L+M+N+O)-(J+O)-(E).

The financial trader surplus is –(K+L+M+N). DA payment to the financial trader is -V·LMP1′, or –(K+L+M+N+O). The RT balancing payment by the financial trader is V·LMP2′, the area of O. As in Figure 4, DEC are settled as they bid demand. The demand bidders including the financial trader's virtual demand bid pay for energy in the DA market and are paid a balancing payment in the RT market when RT demand is less than DA demand. The total financial trader surplus is the area of –(K+L+M+N), or –(K+L+M+N+O)+O which is negative. Consequently, with the negative RT demand, bidding a DEC would not profit the financial trader in this case.

3.4.4 Graphical Representations: With Virtual Bids, RT demand is greater than DA demand

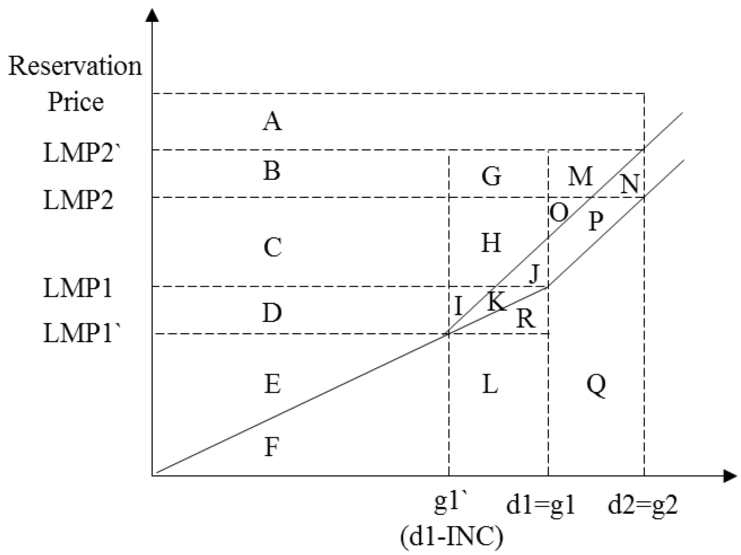

Figure 3.10 Welfare calculation when RT demand is greater than DA demand with an INC

Figure 3.10 is the case when DA demand is lower than RT demand with INC. Without the financial trader, the consumer surplus is A+B+C+G+H+J+M+N, and the producer surplus is D+E+I+K+O+P. With the financial trader, the consumer surplus is A+B+C+D+G+H+I+J+K+R, the producer surplus is E+G+H+I+M+O and financial trader surplus is –(G+H+I+J+K+R).

Calculating economic surplus without INC, the DA payment by the consumer is d1·LMP1 or the area of D+E+F+I+K+R+L and the RT balancing payment by the consumer is (d1-d2)·LMP2 or the area of –(O+P+Q) when d2 is larger than d1. The consumer value is d2·RP or the area of A+B+C+D+E+F+G+H+I+J+K+L+M+N+O+P+Q. Thus, the total consumer surplus is the area of A+B+C+G+H+J+M+N, or -(D+E+F+I+K+R+L)-(O+P+Q) +(A+B+C+D+E+F+G+H+I+J+K+L+M+N+O+P+Q+R).

DA payment to the producer is g1·LMP1 or the area of D+E+F+I+K+R+L. The RT balancing payment to the producer is -(g1-g2)·LMP2, the area of O+P+Q when g2 is larger than g1, and the generation cost is the area under the inverse supply curve up to d2, F+R+L+Q. The total producer surplus is the area of D+E+I+K+O+P, or (D+E+F+I+K+R+L) +(O+P+Q) - (F+R+L+Q).

With the INC, the cleared generation in DA (g1´) must equal the overall bid demand in DA (g1´ = d1 – INC), which is the physical demand bid less the virtual supply offer (INC). Bidding virtual supply makes the overall generation requirement smaller than physical demand bid. Cleared generation in DA with an INC bid (g1´) sets the new DA market price (LMP1´), lower than DA market price without the INC (LMP1).

When RT generation is greater than the cleared generation in the DA market, the price is set based on the steeper portion of the RT inverse supply curve. Both DA and RT inverse supply curves intersect at the cleared generation and price in DA, and the RT inverse supply curve is steeper than DA inverse supply curve. Hence the level of cleared generation in the DA market is influenced by the INC, changing the RT market price. As demonstrated in figure 3.10, without the INC, the kink of the supply curve occurs at g1 so the LMP2 would be set along the lower RT supply curve to the right of g1.

Given the changes in DA generation and clearing price due to the INC, the DA payment by the consumer is d1·LMP1´, or the area of E+F+L, and the RT balancing payment by the consumer is (d1-d2)·LMP2´, or –(M +N+O+P+Q) when d2 is larger than d1. The consumer value is d2·RP or the area of A+B+C+D+E+F+G+H+I+J+K+L+M+N+O+P+Q+R, and thus, the total consumer surplus is the area of A+B+C+D+G+H+I+J+K+R, or -(E+F+L)-(M+N+O+P+Q)+(A+B+C+D+E+F+G+H+I+J+K+L+M+N+O+P+Q+R).

The DA payment to the producer is g1´·LMP1´ or the area of E+F, and the RT balancing payment to the producer is -(g1´-g2)·LMP2´, or the area of G+H+I+J+K+R+L+M+N+O+P+Q when g2 is less than g1´. The generation cost is the area under the inverse RT supply curve up to d2 or F+J+K+R+L+N+P+Q. Total producer surplus is the area of E+G+H+I+M+O, or (E+F)+(G+H+I+J+K+R+L+M+N+O+P+Q)-(F+J+K+R+L+N+P+Q).

The financial trader surplus is –(G+H+I+J+K+R), which is negative in this case. The DA payment to the financial trader is V·LMP1´, or the area of L. The RT balancing payment by the financial trader is V·LMP2´, or the area of G+H+I+J+K+R+L. The supply offers including the financial trader's virtual supply offer need to be paid in DA market and power must be purchased in the RT market. The total financial trader surplus is the area of -(G+H+I+J+K+R), or (L)-(G+H+I+J+K+R+L) which is negative. Consequently, bidding an INC would not be profitable for the financial trader in this situation.

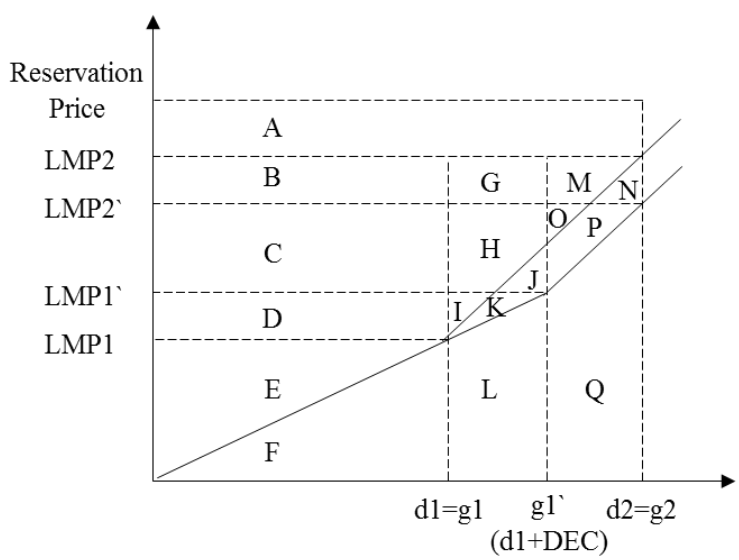

Figure 3.11 Welfare calculation when RT demand is greater than DA demand with a DEC

Figure 3.11 is the case when DA demand is lower than RT demand with a DEC making the committed generation in DA greater than DA demand and less than RT demand. Without the financial trader, the consumer surplus is A+B+C+D, and the producer surplus is E+G+H+I+M+O. With the financial trader, the consumer surplus is A+B+C+G+M+N, the producer surplus is D+E+I+K+O+P, and the financial trader surplus is H+J.

Calculating economic surplus without the DEC, the DA payment by the consumer is d1·LMP1, the area of E+F and the RT balancing payment by the consumer is (d1-d2)·LMP2 or the area of –(G+H+I+J+K+L+M+N+O+P+Q) when d2 is larger than d1. The consumer value is d2·RP or the area of A+B+C+D+E+F+G+H+I+J+K+L+M+N+O+P+Q. Thus, the total consumer surplus is the area of A+B+C+D, or –(E+F)-(G+H+I+J+K+L+M+N+O+P+Q) +(A+B+C+D+E+F+G+H+I+J+K+L+M+N+O+P+Q).

The DA payment to the producer is g1·LMP1 or the area of E+F and the RT balancing payment to the producer is –(g1-g2)·LMP2, or the area of G+H+I+J+K+L+M+N+O+P+Q when g2 is larger than g1, and the generation cost is the area under the inverse supply curve up to d2 or F+J+K+L+N+P+Q. Thus, total producer surplus is the area of E+G+H+I+M+O, or (E+F)+(G+H+I+J+K+L+M+N+O+P+Q)-(F+J+K+L+N+P+Q).

With the DEC bid, the DA payment by the consumer is d1·LMP1′, or the area D+E+F, and the RT balancing payment to the consumer is (d1-d2)·LMP2′, or the area of –(H+I+J+K+L+O+P+Q) when d2 is larger than d1. Since the consumer value is d2·RP, or the area A+B+C+D+E+F+G+H+I+J+K+L+M+N+O+P+Q+R, total consumer surplus is the area of A+B+C+G+M+N, or -(D+E+F)-(H+I+J+K+L+O+P+Q) +(A+B+C+D+E+F+G+H+I+J+K+L+M+N+O+P+Q+R).

Again with the DEC, the DA payment to the producer is g1´·LMP1´ or the area of D+E+F+I+K+L, and the RT balancing payment to the producer is -(g1´-g2)·LMP2´, or the area of O+P+Q when g2 is greater than g1´. The generation cost is the area under the RT inverse supply curve up to d2 or F+L+Q. Thus, the total producer surplus is the area of D+E+I+K+O+P, or (D+E+F+I+K+L)-(O+P+Q)-(F+L+Q).

The financial trader surplus is H+J. The DA payment to the financial trader is V·LMP1´, the area of I+K+L. RT payment by the financial trader is V·LMP2´, the area of H+I+J+K+L. The demand bids including the financial trader's virtual demand bid pay into the DA market and have balancing payments in the RT market. The total financial trader surplus is the area of H+J, or –(I+K+L)+(H+I+J+K+L) which is positive. Consequently, bidding a DEC and making the committed generation in DA greater than DA demand and less than RT demand would be profitable for the financial trader.

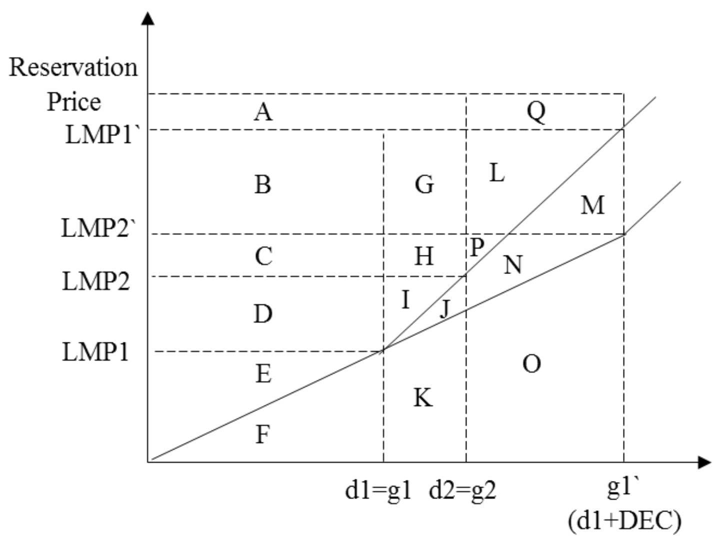

Figure 3.12 Welfare calculation when RT demand is greater than DA demand with a DEC

Figure 3.12 is the case when DA demand is lower than RT demand with a DEC making the committed generation in the DA market greater than RT demand. Without the financial trader, the consumer surplus is A+B+C+D+G+H, and the producer surplus is E+I. With the financial trader, the consumer surplus is A+G the producer surplus is B+C+D+E+G+H+I+J+L+M and the financial trader loss is G+L+M.

Calculating economic surplus without the DEC, the DA payment by the consumer is d1·LMP1 or the area of E+F and the RT balancing payment by the consumer is (d1-d2)·LMP2 or the area of –(I+J+K) when d2 is larger than d1. The consumer value is d2·RP or the area of A+B+C+D+E+F+G+H+I+J+K. Thus, the total consumer surplus is the area of A+B+C+D+G+H, or -(E+F)-(I+J+K)+(A+B+C+D+E+F+G+H+I+J+K). The DA payment to the producer is g1·LMP1 or the area of E+F and the RT balancing payment to the producer is -(g1-g2)·LMP2 or the area of I+J+K when g2 is larger than g1. The generation cost is the area under the inverse supply curve up to d2 or F+J+K. Thus, the total producer surplus is the area of E+I, or (E+F)+(I+J+K)-(F+J+K).

Based on the changed generation and price by the DEC, the DA payment by the consumer is d1·LMP1´, or the area of B+C+D+E+F, and the RT balancing payment to the consumer is (d1-d2)·LMP2´, or the area of –(H+I+J+K) when d2 is larger than d1. Since the consumer value is d2·RP or the area of A+B+C+D+E+F+G+H+I+J+K, the total consumer surplus is the area of A+G, or -(B+C+D+E+F)-(H+I+J+K)+(A+B+C+D+E+F+G+H+I+J+K).

DA payment to the producer is g1´·LMP1´ or the area of B+C+D+E+F+G+H+I+J+K+L+M+N+O+P, and the RT balancing payment to the producer is -(g1´-g2)·LMP2´, or the area of –(N+O+P) when g2 is less than g1´. The generation cost is the area under the RT inverse supply curve up to d2 or F+K. Thus the total producer surplus is the area of B+C+D+E+G+H+I+J+L+M, or (B+C+D+E+F+G+H+I+J+K+L+M+N+O+P)-(N+O+P)-(F+K).

The financial trader surplus is –(G+L+M) in this case. The DA payment to the financial trader is V·LMP1′, or the area of G+H+I+J+K+L+M+N+O+P. The RT balancing payment by the financial trader is V·LMP2′, or the area of H+I+J+K+N+O+P. The total financial trader surplus is the area of –(G+L+M), or

-(G+H+I+J+K+L+M+N+O+P) +(+I+J+K+N+O+P) which is negative. Consequently, with positive RT demand deviation, the financial trader would not profit from bidding a DEC.

Each graph presented above indicates the welfare calculations with and without virtual bidding for various market situations that include: RT demand greater or less then DA demand, the virtual bid being an INC or DEC, and the magnitude of the virtual bid being greater or less than the change in demand between the RT and DA markets.

The financial trader would like to employ an optimal bidding strategy, that is one that maximizes the expected profit based on the distribution of the demand deviation and other market parameters. Because the virtual transactions shift the supply curve due to the differences in the slopes of the supply curves in the DA and RT markets, financial traders can achieve a positive expected profit.

3.5 Expected Welfare Change of Market Participants due to Virtual Transactions

The RT demand, d2, is the sum of DA demand, d1, and demand deviation Δ which has probability distribution function f(Δ). Here we assume the distribution of Δ is uniform with support [-α+μ, α+μ] where 'α' represents the range of uncertainty and 'μ' represents the mean of the uniform distribution.

The probabilistic nature of RT demand influences the RT generation and RT price as a consequence. The expected RT price without virtual bids, $\mathrm{E}[\mathrm{P2}(\mathrm{d1}+\Delta)]$, is the sum of the integral of the DA inverse supply curve(P1) up to Δ=0 plus the integral of the RT inverse supply curve(P2) over the range of positive Δ.

With virtual bid case, the expected RT price is:

$$E[P2'(d1+\Delta))] = \frac{1}{2\alpha}\left\{\int_{-\alpha+\mu}^{-V} P1(d1+\Delta)d\Delta + \int_{-V}^{\alpha+\mu} P2'(d1+\Delta)d\Delta\right\}$$

Without a virtual bid, the expected RT price is:

$$E[P2(d1+\Delta))] = \frac{1}{2\alpha}\left\{\int_{-\alpha+\mu}^{0} P1(d1+\Delta)d\Delta + \int_{0}^{\alpha+\mu} P2(d1+\Delta)d\Delta\right\}$$

The first integral represents payments for a negative deviation, and thus the RT price is set by the DA inverse supply curve. The second integral represents payments for the positive deviations, and thus the RT price is set by the RT inverse supply curve. The amount of virtual bid (note that V is positive for an INC and negative for a DEC) shifts the intersection of the DA supply and the RT supply curve thus the boundaries where the RT price is determined.

3.5.1 Financial Trader Welfare with the Optimal Virtual Bid

The expected financial trader surplus is,

$$F(V) = E[V(LMP1' - LMP2')] = V(P1(d1-V) - E[P2'(d1+\Delta)])$$

$$= V\left[-\frac{a1}{b1} + \frac{d1-V}{b1} - \frac{1}{2\alpha}\left\{\int_{-\alpha+\mu}^{-V} P1(d1+\Delta)d\Delta + \int_{-V}^{\alpha+\mu} P2'(d1+\Delta)d\Delta\right\}\right]$$

$$= V\left(\frac{1}{4}\frac{-(\alpha+\mu+V)^2 b1 + b2(-\alpha+\mu+V)^2}{b2}\right)$$

Differentiating the expected profit for financial trader w.r.t V yields the following:

$$F'(V) = \frac{3b2(-\frac{\alpha}{3}+\frac{\mu}{3}+V)(-\alpha+\mu+V) - 3b1(\frac{\alpha}{3}+\frac{\mu}{3}+V)(\alpha+\mu+V)}{4\alpha b1 b2}$$

The optimal bidding strategy (V^*) for the financial trader is found by setting this derivative to zero and solving for the bid. This derivative has two roots:

$$V = \frac{1}{3(b1-b2)}\left(-2b1\alpha - 2b2\alpha - 2\mu b1 + 2\mu b2 + \sqrt{\alpha^2 b1^2 + 14\alpha^2 b1b2 + \alpha^2 b2^2 + 2\alpha b1^2\mu - 2\alpha b2^2\mu + b1^2\mu^2 - 2b1b2\mu^2 + b2^2\mu^2}\right)$$

and

$$V = -\frac{1}{3(b1-b2)}\left(2b1\alpha + 2b2\alpha + 2\mu b1 - 2\mu b2 + \sqrt{\alpha^2 b1^2 + 14\alpha^2 b1b2 + \alpha^2 b2^2 + 2\alpha b1^2\mu - 2\alpha b2^2\mu + b1^2\mu^2 - 2b1b2\mu^2 + b2^2\mu^2}\right)$$

The former is the optimal bidding strategy because the latter is always negative as shown in Lemma 4. Hence, the optimal bidding strategy V^* is:

$$V^* = \frac{1}{3(b1-b2)}\left(-2b1\alpha - 2b2\alpha - 2\mu b1 + 2\mu b2 + \sqrt{\alpha^2 b1^2 + 14\alpha^2 b1b2 + \alpha^2 b2^2 + 2\alpha b1^2\mu - 2\alpha b2^2\mu + b1^2\mu^2 - 2b1b2\mu^2 + b2^2\mu^2}\right)$$

There are two distinct strategies for the virtual trader. If μ is below a critical value (δ^*), then bidding a positive V (an INC) is optimal while μ is above a critical value, bidding negative V (a DEC) is optimal. The critical value, δ^*, is a function of α, b1, and b2 that is $-\alpha\frac{\sqrt{b1}-\sqrt{b2}}{\sqrt{b1}+\sqrt{b2}}$, which is negative. The welfare impacts on the consumer and producer are based on these two types of virtual bidding strategy.

1. If $\mu < \delta^*, V^* > 0$ (an INC) is optimal bidding strategy
2. If $\mu > \delta^*, V^* < 0$ (a DEC) is optimal bidding strategy

If the expected demand deviation is zero, μ is greater than δ^* ($\mu = 0 > \delta^*$), hence bidding DECs ($V^* < 0$) is the optimal strategy for the financial trader. The optimal bidding strategy when $\mu = 0$ is:

$$V^* = \frac{\alpha}{3(b1-b2)}\left(-2b1 - 2b2 + \sqrt{b1^2 + 14b1b2 + b2^2}\right)$$

Lemma 1. $-2(b1 + b2) + \sqrt{b1^2 + 14b1b2 + b2^2} < 0$ when b1, b2, and α > 0, and b1 > b2.

Proof.

Clearly if b1 ≠ b2, then $3(b1-b2)^2>0$. Expanding this expression yields $3b1^2-6b1b2+3b2^2>0$, and adding the positive expression $b1^2+14b1b2+b2^2$ on both sides yields $4b1^2+8b1b2+4b2^2>b1^2+14b1b2+b2^2$. Taking the square root of both sides yields $2(b1+b2)>\sqrt{b1^2+14b1b2+b2^2}$, and rearranging that we get $-2(b1+b2)+\sqrt{b1^2+14b1b2+b2^2}<0$.

Lemma 2. If μ is below a critical value ($\delta^*=-\alpha\frac{\sqrt{b1}-\sqrt{b2}}{\sqrt{b1}+\sqrt{b2}}$), bidding a positive V (an INC) is optimal, and when μ is above the critical value, and bidding a negative V (a DEC) is optimal.

Proof.

Assume $-\alpha\frac{\sqrt{b1}-\sqrt{b2}}{\sqrt{b1}+\sqrt{b2}}=\delta^*<\mu$. Rearranging it yields $-\alpha\sqrt{b1}+\alpha\sqrt{b2}<\mu\sqrt{b1}+\mu\sqrt{b2}$, $\sqrt{b1}(-\alpha-\mu)<\sqrt{b2}(-\alpha+\mu)$, and $\sqrt{b1}(\alpha+\mu)>\sqrt{b2}(\alpha-\mu)$. Squaring both sides yields $b1(\alpha+\mu)^2>b2(\alpha-\mu)^2$, and $b1(\alpha+\mu)^2-b2(\alpha-\mu)^2>0$. Multiplying $3(b1-b2)$ yields $3(b1-b2)(-b2(\alpha-\mu)^2+b1(\alpha+\mu)^2)>0$

Expanding the previous inequality yields

$$3\alpha^2b1^2-6\alpha^2b1b2+3\alpha^2b2^2+6\alpha b1^2\mu-6\alpha b2^2\mu+3b1^2\mu^2-6b1b2\mu^2+3b2^2\mu^2$$

$$=(4\alpha^2b1^2+8\alpha^2b1b2+4\alpha^2b2^2+8\alpha b1^2\mu-8\alpha b2^2\mu+4b1^2\mu^2-8b1b2\mu^2+4b2^2\mu^2)-$$

$$(\alpha^2b1^2+14\alpha^2b1b2+\alpha^2b2^2+2\alpha b1^2\mu-2\alpha b2^2\mu+b1^2\mu^2-2b1b2\mu^2+b2^2\mu^2)$$

$$=(2\alpha(b1+b2)+2\mu(b1-b2))^2-$$

$$\left(\sqrt{\alpha^2b1^2+14\alpha^2b1b2+\alpha^2b2^2+2\alpha b1^2\mu-2\alpha b2^2\mu+b1^2\mu^2-2b1b2\mu^2+b2^2\mu^2}\right)^2>0.$$

Moving the squared root term to the other side and rearranging it yields

$$-(-2b1\alpha-2b2\alpha-2\mu b1+2\mu b2)>$$

$$\sqrt{\alpha^2b1^2+14\alpha^2b1b2+\alpha^2b2^2+2\alpha b1^2\mu-2\alpha b2^2\mu+b1^2\mu^2-2b1b2\mu^2+b2^2\mu^2}),$$

which can be rearranged as

$-2b1\alpha - 2b2\alpha - 2\mu b1 + 2\mu b2 +$

$\sqrt{\alpha^2 b1^2 + 14\alpha^2 b1b2 + \alpha^2 b2^2 + 2\alpha b1^2\mu - 2\alpha b2^2\,\mu + b1^2\,\mu^2 - 2b1b2\mu^2 + b2^2\mu^2} < 0.$

Multiplying both sides by $\frac{1}{3(b1-b2)} > 0$ yields

$\frac{-2b1\alpha - 2b2\alpha - 2\mu b1 + 2\mu b2 + \sqrt{\alpha^2 b1^2 + 14\alpha^2 b1b2 + \alpha^2 b2^2 + 2\alpha b1^2\mu - 2\alpha b2^2\,\mu + b1^2\,\mu^2 - 2b1b2\mu^2 + b2^2\mu^2}}{3(b1-b2)} = V^* < 0.$

Hence, when μ is above the critical value ($\delta^* = -\alpha\frac{\sqrt{b1}-\sqrt{b2}}{\sqrt{b1}+\sqrt{b2}}$), bidding negative V (a DEC) is the optimal strategy. Bidding positive V (an INC) is, analogously the optimal bidding strategy when μ is above the critical value δ^*.

Lemma 3. The critical value ($\delta^* = -\alpha\frac{\sqrt{b1}-\sqrt{b2}}{\sqrt{b1}+\sqrt{b2}}$), is getting smaller when the uncertainty, α, is getting larger. Also, the critical value is getting smaller when relative steepness is greater.

Proof.

Clearly if b1 > b2, then $\frac{\sqrt{b1}-\sqrt{b2}}{\sqrt{b1}+\sqrt{b2}} > 0$. Since α>0, the critical value $-\alpha\frac{\sqrt{b1}-\sqrt{b2}}{\sqrt{b1}+\sqrt{b2}}$ is negative and getting smaller when α is getting greater.

The critical value $-\alpha\frac{\sqrt{b1}-\sqrt{b2}}{\sqrt{b1}+\sqrt{b2}}$, becomes $-\alpha(\frac{b1+b2-2\sqrt{b1b2}}{b1-b2})$ by multiplying $\sqrt{b1}-\sqrt{b2}$ to the numerator and denominator. Assume that steepness of RT slope, $s = \frac{b2}{b1}$, rearranging the critical value yields $-\alpha\left(\frac{b2(s+1)-2b2\sqrt{s}}{b2(s-1)}\right) = -\alpha b2\left(\frac{s+1-2\sqrt{s}}{s-1}\right)$. Fixing α and b2, when the relative steepness, s, is getting greater, the critical value is getting smaller.

Lemma 4. The optimal bidding strategy (V^*) for the financial trader is $V^* = \frac{1}{3(b1-b2)}(-2b1\alpha -$

$2b2\alpha - 2\mu b1 + 2\mu b2 +$

$\sqrt{\alpha^2 b1^2 + 14\alpha^2 b1b2 + \alpha^2 b2^2 + 2\alpha b1^2\mu - 2\alpha b2^2\,\mu + b1^2\,\mu^2 - 2b1b2\mu^2 + b2^2\mu^2})$

since $V^{**} = \frac{1}{3(b1-b2)}(-2b1\alpha - 2b2\alpha - 2\mu b1 + 2\mu b2 +$

$\sqrt{\alpha^2 b1^2 + 14\alpha^2 b1b2 + \alpha^2 b2^2 + 2\alpha b1^2\mu - 2\alpha b2^2\mu + b1^2\mu^2 - 2b1b2\mu^2 + b2^2\mu^2})$

is always negative when b1, b2, and $\alpha > 0$.

Proof.

Differentiating the expected profit for financial trader w.r.t V yields $F'(V) =$

$\frac{3b2(-\frac{\alpha}{3}+\frac{\mu}{3}+V)(-\alpha+\mu+V)-3b1(\frac{\alpha}{3}+\frac{\mu}{3}+V)(\alpha+\mu+V)}{4\alpha b1b2}$ and $F''(V) = \frac{(-3V-2\alpha-2\mu)b1+3b2(V-\frac{2\alpha}{3}+\frac{2\mu}{3})}{2\alpha b1b2}$.

Plug in V^* to $F''(V)$, $F''(V^*)$ yields $-\frac{\sqrt{(\alpha+\mu)^2 b1^2+(14\alpha^2-2\mu^2)b1b2+(\alpha-\mu)^2 b2^2}}{2\alpha b1b2}$ which is negative.

Plug in V^{**} to $F''(V)$, $F''(V^{**})$ yields $\frac{\sqrt{(\alpha+\mu)^2 b1^2+(14\alpha^2-2\mu^2)b1b2+(\alpha-\mu)^2 b2^2}}{2\alpha b1b2}$ which is positive.

Hence, $V^{**}$is always negative.

Theorem 1. The optimal virtual bid is negative (a DEC) with zero expected demand deviation.

Proof.

$V^* = \frac{1}{3(b1-b2)}\left(-2b1\alpha - 2b2\alpha + \sqrt{\alpha^2 b1^2 + 14\alpha^2 b1b2 + \alpha^2 b2^2}\right)$.

Factoring out α from the terms in the parenthesis yields

$\frac{1}{3(b1-b2)}\left(-2\alpha(b1+b2) + \alpha\sqrt{b1^2 + 14b1b2 + b2^2}\right)$

$= \frac{1}{3(b1-b2)}\alpha(\sqrt{b1^2 + 14b1b2 + b2^2} - 2(b1+b2))$.

From Lemma 1, $\sqrt{b1^2 + 14b1b2 + b2^2} < 2(b1+b2)$, and so the term in the parenthesis in the equation is negative, $(\sqrt{b1^2 + 14b1b2 + b2^2} - 2(b1+b2)) < 0$. Since $\alpha>0$ and $b1 > b2 > 0$, $\frac{1}{3(b1-b2)} > 0$, $V^* = \frac{1}{3(b1-b2)}\alpha(-2b1 - 2b2 + \sqrt{b1^2 + 14b1b2 + b2^2})$ is negative. Hence, V^* is negative (a DEC is optimal) when $\mu = 0$.

3.5.2 Consumer Welfare Change with the Optimal Virtual Bid

The difference in the DA payment by the consumer is the negative of DA demand times DA price differences due to virtual bids.

Δ DA Payment by the consumer

$$= -\mathrm{d1} \cdot (\mathrm{LMP1}' - \mathrm{LMP1})$$

$$= -\mathrm{d1} \cdot (\mathrm{P1}(\mathrm{d1} - \mathrm{V}^*) - \mathrm{P1}(\mathrm{d1}))$$

$$= -\mathrm{d1} \cdot \left\{-\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{d1}-\mathrm{V}^*}{\mathrm{b1}} - \left(-\frac{\mathrm{a1}}{\mathrm{b1}} + \frac{\mathrm{d1}}{\mathrm{b1}}\right)\right\}$$

$$= -\mathrm{d1} \cdot \left(-\frac{\mathrm{V}^*}{\mathrm{b1}}\right)$$

The differences in expected RT balance payment by the consumer with V^* are the following:

Δ RT balance payment by the consumer

$$= \mathrm{E}[-\Delta \cdot \mathrm{P2}'(\mathrm{d1} + \Delta)] - \mathrm{E}[-\Delta \cdot \mathrm{P2}(\mathrm{d1} + \Delta)]$$

$$= \frac{1}{2\alpha}\left\{\int_{-\alpha+\mu}^{-\mathrm{V}} -\Delta \cdot \mathrm{P1}(\mathrm{d1} + \Delta)\mathrm{d}\Delta + \int_{-\mathrm{V}}^{\alpha+\mu} -\Delta \cdot \mathrm{P2}'(\mathrm{d1} + \Delta)\mathrm{d}\Delta\right\} - \frac{1}{2\alpha}\left\{\int_{-\alpha+\mu}^{0} -\Delta \cdot \mathrm{P1}(\mathrm{d1} + \Delta)\mathrm{d}\Delta + \int_{0}^{\alpha+\mu} -\Delta \cdot \mathrm{P2}(\mathrm{d1} + \Delta)\mathrm{d}\Delta\right\}$$

$$= \frac{\mathrm{V}^*(\mathrm{b1}-\mathrm{b2})\left\{-3(\alpha+\mu)^2 + \mathrm{V}^{*2}\right\}}{12\alpha\mathrm{b1b2}}$$

The expected overall consumer welfare difference is the sum of differences in DA payment by the consumer and RT balancing payments by the consumer.

Δ Consumer welfare

$$= -\mathrm{d1} \cdot \left(-\frac{\mathrm{V}^*}{\mathrm{b1}}\right) + \frac{\mathrm{V}^*(\mathrm{b1}-\mathrm{b2})(-3(\alpha+\mu)^2 + \mathrm{V}^{*2})}{12\alpha\mathrm{b1b2}}$$

When the expected demand deviation is zero, μ=0, the overall consumer welfare difference becomes:

$$= -\mathrm{d1} \cdot \left(-\frac{\mathrm{V}^*}{\mathrm{b1}}\right) + \frac{\mathrm{V}^*(\mathrm{b1}-\mathrm{b2})(-3\alpha^2 + \mathrm{V}^{*2})}{12\alpha\mathrm{b1b2}}$$

$$= \frac{1}{36\alpha b1b2}\left(-2\alpha b1 - 2\alpha b2 + \sqrt{\alpha^2 b1^2 + 14\alpha^2 b1b2 + \alpha^2 b2^2}\right)\left(-3\alpha^2 + \frac{\left(-2\alpha b1 - 2\alpha b2 + \sqrt{\alpha^2 b1^2 + 14\alpha^2 b1b2 + \alpha^2 b2^2}\right)^2}{9(b1-b2)^2}\right) + \frac{\left(-2\alpha b1 - 2\alpha b2 + \sqrt{\alpha^2 b1^2 + 14\alpha^2 b1b2 + \alpha^2 b2^2}\right)d1}{3b1(b1-b2)}.$$

Theorem 2. Optimal virtual bid generally decreases consumer welfare with zero expected demand deviation when the slope in RT is not substantially steeper than the slope in DA, $\frac{b1-b2}{b2} < \frac{18d1}{5\alpha}$, rearranged as $\frac{b2}{b1} > \frac{5x}{1800+5x}$, $when\ \alpha = x\%$ of d1. For example, the consumer welfare decreases when the slope in RT is not greater than 37 times steeper than the slope in DA when the range of uncertainty is 10% of DA demand.

Proof.

The expected consumer welfare change is $-d1\left(-\frac{V^*}{b1}\right) + \frac{V^*(b1-b2)\left(-3\alpha^2 + V^{*2}\right)}{12\alpha b1b2}$. Factor out V^* and $12\alpha b2d1$ yields $V^* 12\alpha b2d1\left\{(b1-b2)\left(-3\alpha^2 + V^{*2}\right) + 12\alpha b2d1\right\}$. Replacing V^* in the bracket to the expansion form yields $V^* 12\alpha b2d1\left[(b1-b2)\left\{-3\alpha^2 + \frac{\left(-2\alpha b1 - 2\alpha b2 + \sqrt{\alpha^2 b1^2 + 14\alpha^2 b1b2 + \alpha^2 b2^2}\right)^2}{9(b1-b2)^2}\right\} + 12\alpha b2d1\right]$. Expanding the terms in the square bracket yields $V^* 12\alpha b2d1\left[-\frac{\alpha}{9(b1-b2)} 22\left\{\alpha b1^2 + \frac{2}{11}\alpha(b1+b2)\sqrt{b1^2 + 14b1b2 + b2^2} - \frac{38}{11}b1b2\left(\alpha + \frac{27d1}{19}\right) + b2^2\left(\alpha + \frac{54d1}{11}\right)\right\}\right]$.

The terms in the curly bracket, $22\left\{\alpha b1^2 + \frac{2}{11}\alpha(b1+b2)\sqrt{b1^2 + 14b1b2 + b2^2} - \frac{38}{11}b1b2\left(\alpha + \frac{27d1}{19}\right) + b2^2\left(\alpha + \frac{54d1}{11}\right)\right\}$ can be rearranged to $22\alpha b1^2 + 4\alpha(b1 + b2)\sqrt{b1^2 + 14b1b2 + b2^2} + 22b2^2\left(\alpha + \frac{54d1}{11}\right) - 2b1b2(38\alpha + 54d1)$, and rearranged again as $4\alpha(b1 + b2)\sqrt{b1^2 + 14b1b2 + b2^2} + 22\alpha b1^2 - 76\alpha b1b2 + 22\alpha b2^2 - 108b1b2d1 + 108b2^2 d1$, which makes

$V^{*}12\alpha b2d1\left[-\frac{\alpha}{9(b1-b2)}\left\{4\alpha(b1+b2)\sqrt{b1^2+14b1b2+b2^2}+22\alpha b1^2-76\alpha b1b2+\right.\right.$

$\left.\left.22\alpha b2^2-108b1b2d1+108b2^2d1\right\}\right]$.

From Lemma 1, the term in the curly bracket, $4\alpha(b1+b2)\sqrt{b1^2+14b1b2+b2^2}+22\alpha b1^2-$

$76\alpha b1b2+22\alpha b2^2-108b1b2d1+108b2^2d1,$ is less than,

$4\alpha(b1+b2)2(b1+b2)+22\alpha b1^2-76\alpha b1b2+22\alpha b2^2-108b1b2d1+108b2^2d1$

$=8\alpha(b1^2+14b1b2+b2^2)+22\alpha b1^2-76\alpha b1b2+22\alpha b2^2-108b1b2d1+$

$108b2^2d18\alpha b1^2+16\alpha b1b2+8\alpha b2^2+22\alpha b1^2-76\alpha b1b2+22\alpha b2^2-108b1b2d1+$

$108b2^2d1$

$=30\alpha b1^2-60\alpha b1b2+30\alpha b2^2-108b1b2d1+108b2^2d1$, which can be rearranged as

$30(b1-b2)\left\{\alpha b1+b2\left(-\alpha-\frac{18d1}{5}\right)\right\}$.

The term in the square bracket $\alpha b1+b2\left(-\alpha-\frac{18d1}{5}\right)$ is negative based on the assumption

$\frac{b1-b2}{b2}<\frac{18d1}{5\alpha}$, making an inequality relationship that $\left\{4\alpha(b1+b2)\sqrt{b1^2+14b1b2+b2^2}+\right.$

$\left.22\alpha b1^2-76\alpha b1b2+22\alpha b2^2-108b1b2d1+108b2^2d1\right\}<30(b1-b2)\left\{\alpha b1+\right.$

$\left.b2\left(-\alpha-\frac{18d1}{5}\right)\right\}<0$.

From Theorem 2, $V^{*}<0$, and $V^{*}12\alpha b2d1\left[-\frac{\alpha}{9(b1-b2)}\left\{4\alpha(b1+\right.\right.$

$b2)\sqrt{b1^2+14b1b2+b2^2}+22\alpha b1^2-76\alpha b1b2+22\alpha b2^2-108b1b2d1+$

$\left.\left.108b2^2d1\right\}\right]$ becomes negative. Hence, the consumer welfare change is negative then the optimal virtual bid with zero expected demand deviation when the slope of RT is not substantially steeper than the slope in DA, $\frac{b1-b2}{b2}<\frac{18d1}{5\alpha}$.

3.5.3 Producer Welfare Change with the Optimal Virtual Bid

Producer surplus differences are from DA payment, RT balance payment, and generation cost. The differences in the DA payment to the producer is:

Δ DA Payment to the producer

$$= g1' \cdot LMP1' - g1 \cdot LMP1$$

$$= (d1 - V^*) \cdot P1(d1 - V^*) - d1 \cdot P1(d1)$$

$$= \frac{V^*(V^* + a1 - 2d1)}{b1}$$

The expected differences in the RT balancing payments to the producer are the changes in the differences between RT and DA generation times RT prices by virtual bid.

Δ RT Payment to the producer

$$= (\Delta + V^*)LMP2' - \Delta \cdot LMP2$$

$$= E[(\Delta + V^*) \cdot P2'(d1 + \Delta)] - E[\Delta \cdot P2(d1 + \Delta)]$$

$$= \frac{1}{2\alpha}\left[\int_{-\alpha+\mu}^{-V^*}(\Delta + V^*) \cdot P1(d1 + \Delta)d\Delta + \int_{-V^*}^{\alpha+\mu}(\Delta + V^*) \cdot P2'(d1 + \Delta)d\Delta\right] -$$

$$\frac{1}{2\alpha}\left\{\int_{-\alpha+\mu}^{0}\Delta \cdot P1(d1 + \Delta)d\Delta + \int_{0}^{\alpha+\mu}\Delta \cdot P2(d1 + \Delta)d\Delta\right\}$$

$$= \frac{V^*\left[b2\left\{-3\alpha^2 - 3\mu^2 + \alpha(-6a1 + 6d1 - 3V^*) - 3\mu V^* - V^{*2}\right\} + b1\left\{3\alpha^2 + 3\mu^2 + 3\mu V^* + V^{*2} + \alpha(6\mu + 3V^*)\right\}\right]}{6\alpha b1 b2}$$

The expected difference in generation cost is the change in the area under the DA and RT supply curves.

Δ Generation cost

$$= -\frac{1}{2\alpha}\left[\int_{\mu-\alpha}^{-V^*}\left(\int_0^{d1+\Delta}P1(g)dg\right)d\Delta + \int_{-V^*}^{\mu+\alpha}\left\{\int_0^{d1-V^*}P1(g)\,dg + \int_{d1-V^*}^{d1+\Delta}P2'(g)\,dg\right\}d\Delta\right] +$$

$$\frac{1}{2\alpha}\left[\int_{\mu-\alpha}^{0}\left(\int_0^{d1}P1(g)dg\right)d\Delta + \int_0^{\mu+\alpha}\left\{\int_0^{d1}P1(g)\,dg + \int_{d1}^{d1+\Delta}P2(g)\,dg\right\}d\Delta\right]$$

$$= -\frac{1}{12}\frac{(b1 - b2)V^*\left\{V^{*2} + (3\alpha + 3\mu)V^* + 3(\alpha + \mu)^2\right\}}{\alpha b1 b2}$$

The expected producer welfare difference is the sum of DA payment to the producer, RT balance payment to the producer and generation cost.

Δ Producer welfare

$$= \frac{V^*(V^*+a1-2d1)}{b1} +$$

$$\frac{V^*\left[b2\left\{-3\alpha^2-3\mu^2+\alpha(-6a1+6d1-3V^*)-3mV^*-{V^*}^2\right\}+b1\left\{3\alpha^2+3\mu^2+3\mu V^*+{V^*}^2+\alpha(6\mu+3V^*)\right\}\right]}{6\alpha b1b2} -$$

$$\frac{1}{12}\frac{(b1-b2)V^*\left\{{V^*}^2+(3\alpha+3\mu)V^*+3(\alpha+\mu)^2\right\}}{\alpha b1b2}$$

When the expected demand deviation is zero, μ =0, the expected producer welfare is:

$$= \frac{1}{12\alpha b1b2}\left[12\alpha b2V^*(a1-2d1+V^*)+V^*(-b1+b2)(3\alpha^2+3\alpha V^*+{V^*}^2)+\right.$$

$$\left.2V^*\left\{b2(-3\alpha(\alpha+2a1-2d1)-3\alpha V^*-{V^*}^2)+b1(3\alpha^2+3\alpha V+V^2)\right\}\right]$$

$$= -\frac{1}{81b1(b1-b2)^2b2}7\alpha(b1+b2-\frac{1}{2}\sqrt{b1^2+14b1b2+b2^2})\left\{\alpha b1^2+\frac{5}{14}a(b1+\right.$$

$$\left.\frac{23b2}{5})\sqrt{b1^2+14b1b2+b2^2}-\frac{52}{7}b1b2(a+\frac{27d1}{26})+b2^2(-\frac{11a}{7}+\frac{54d1}{7})\right\}$$

Theorem 3. Optimal virtual bid increases producer welfare with zero expected demand deviation when the slope in RT is not substantially steeper than the slope in DA, $\frac{b1-b2}{b2} < \frac{9d1}{2\alpha}$ rearranged as $\frac{b2}{b1} > \frac{2x}{900+2x}, when\ \alpha = x\%$ of d1. For example, the producer welfare increases when the slope in RT is not greater than 45 times steeper than the slope in DA when the range of uncertainty is 10% of DA demand. This condition is relatively weaker than the one in Theorem 2.

Proof.

The expected producer welfare change is $\frac{1}{12\alpha b1b2}\left[12\alpha b2V^*(a1-2d1+V^*)+V^*(-b1+b2)(3\alpha^2+3\alpha V^*+{V^*}^2)+2V^*\left\{b2(-3\alpha(\alpha+2a1-2d1)-3\alpha V^*-{V^*}^2)+b1(3\alpha^2+3\alpha V+V^2)\right\}\right]$ and the V^* in the square bracket can be factored out as $\frac{V^*}{12\alpha b1b2}\left[12\alpha b2(a1-2d1+V^*)+\right.$

$(-b1+b2)\left(3\alpha^2+3\alpha V^*+V^{*2}\right)+2\left\{b2\left(-3\alpha(\alpha+2a1-2d1)-3\alpha V^*-V^{*2}\right)+\right.$

$\left.b1\left(3\alpha^2+3\alpha V^*+V^{*2}\right)\right\}\Big]$.

The term in the square bracket can be rearranged as the following:

$$\frac{V^*}{12\alpha b1b2}\Big[12\alpha b2(-2d1+V^*)+2b2\left(-3\alpha(\alpha-2d1)-3\alpha V-V^{*2}\right)+2b1\left(3\alpha^2+3\alpha V^*+\right.$$

$$\left.V^{*2}\right)+(-b1+b2)\left(3\alpha^2+3\alpha V^*+V^{*2}\right)\Big]$$

$$=\frac{V^*}{12\alpha b1b2}\Big[6\alpha^2b1+3\alpha^2(-b1+b2)-6\alpha b2(\alpha-2d1)-24\alpha b2d1+\Big(6\alpha b1+6\alpha b2+$$

$$3\alpha(-b1+b2)\Big)V^*+(b1-b2)V^{*2}\Big]$$

$$=\frac{V^*}{12\alpha b1b2}\Big[3\alpha^2b1-3\alpha^2b2-12\alpha b2d1+3\alpha b1V^*+9\alpha b2V^*+b1V^{*2}-b2V^{*2}\Big]$$

$$=\frac{V^*}{12\alpha b1b2}\Big[3\alpha^2b1-3\alpha^2b2-12\alpha b2d1+3\alpha V^*(b1+3b2)+(b1-b2)V^{*2}\Big]$$

$$=\frac{V^*}{12\alpha b1b2}\left[\{(b1-b2)V^*+3\alpha(b1+3b2)\}V^*+3\alpha^2b1-3\alpha^2b2-12\alpha b2d1\right] .$$

Replacing the V^* in the square bracket with the extended form and rearranging the terms yields

$$\frac{V^*}{12\alpha b1b2}\Bigg[\frac{3\left(-2\alpha b1-2\alpha b2+9\alpha(b1+3b2)+\sqrt{\alpha^2b1^2+14\alpha^2b1b2+\alpha^2b2^2}\right)\left(-2\alpha b1-2\alpha b2+\sqrt{\alpha^2b1^2+14\alpha^2b1b2+\alpha^2b2^2}\right)}{9(b1-b2)}+$$

$$3\alpha^2b1-3\alpha^2b2-12\alpha b2d1\Bigg]$$

$$=\frac{V^*}{12\alpha b1b2}\Bigg[\frac{\alpha^2\left(-2b1-2b2+\sqrt{b1^2+14b1b2+b2^2}\right)\left(7b1+25b2+\sqrt{b1^2+14b1b2+b2^2}\right)}{9(b1-b2)}+3\alpha^2b1-3\alpha^2b2-$$

$$12\alpha b2d1\Bigg]$$

$$=\frac{V^*}{12\alpha b1b2}\Bigg[\frac{\alpha^2\left(-2b1-2b2+\sqrt{b1^2+14b1b2+b2^2}\right)\left(7b1+25b2+\sqrt{b1^2+14b1b2+b2^2}\right)}{9(b1-b2)}+$$

$$\frac{(b1-b2)\left(27\alpha^2b1-27\alpha^2b2-108\alpha b2d1\right)}{9(b1-b2)}\Bigg]$$

$$=\frac{V^*}{12\alpha b1b2}\Bigg[\frac{\alpha\Big(14\alpha b1^2-104\alpha b1b2-22\alpha b2^2+\alpha\sqrt{b1^2+14b1b2+b2^2}(5b1+23b2)-108d1b2(b1-b2)\Big)}{9(b1-b2)}\Bigg] .$$

Factoring out the term $\frac{\alpha}{9(b1-b2)}$ in the square bracket yields $\frac{V^*}{12\alpha b1b2}\left[\frac{\alpha}{9(b1-b2)}(14\alpha b1^2 - 104\alpha b1b2 - 22\alpha b2^2 + \alpha\sqrt{b1^2+14b1b2+b2^2}(5b1+23b2) - 108d1b2(b1-b2))\right]$. From Lemma 1, the term in the square bracket $14\alpha b1^2 - 104\alpha b1b2 - 22\alpha b2^2 + \alpha\sqrt{b1^2+14b1b2+b2^2}(5b1+23b2) - 108d1b2(b1-b2) < 14\alpha b1^2 - 104\alpha b1b2 - 22\alpha b2^2 + 2\alpha(b1+b2)(5b1+23b2) - 108d1b2(b1-b2)$

Rearranging the latter term yields

$$14\alpha b1^2 - 104\alpha b1b2 - 22\alpha b2^2 + 2\alpha(b1+b2)(5b1+23b2) - 108d1b2(b1-b2)$$

$$= 14\alpha b1^2 - 104\alpha b1b2 - 22\alpha b2^2 + 10\alpha b1^2 + 56\alpha b1b2 + 46\alpha b2^2 - 108d1b2(b1-b2)$$

$$= 24\alpha b1^2 - 48\alpha b1b2 + 24\alpha b2^2 - 108d1b2(b1-b2)$$

$$= 24\alpha(b1-b2)^2 - 108d1b2(b1-b2)$$

$$= 12(b1-b2)\{2\alpha(b1-b2) - 9d1b2\}$$

From $\frac{b1-b2}{b2} < \frac{9d1}{2\alpha}$, the term in the curly bracket becomes negative and it suggests the inequality relationship that $14\alpha b1^2 - 104\alpha b1b2 - 22\alpha b2^2 + \alpha\sqrt{b1^2+14b1b2+b2^2}(5b1+23b2) - 108d1b2(b1-b2) < 14\alpha b1^2 - 104\alpha b1b2 - 22\alpha b2^2 + 2\alpha(b1+b2)(5b1+23b2) - 108d1b2(b1-b2) = 12(b1-b2)\{2\alpha(b1-b2) - 9d1b2\} < 0$.

From Theorem 1, $\frac{V^*}{12\alpha b1b2}\left[\frac{\alpha}{9(b1-b2)}(14\alpha b1^2 - 104\alpha b1b2 - 22\alpha b2^2 + \alpha\sqrt{b1^2+14b1b2+b2^2}(5b1+23b2) - 108d1b2(b1-b2))\right]$ is positive. Hence, the producer welfare change is positive if the optimal virtual bid is chosen when there is zero expected demand deviation when the slope in RT is not substantially steeper than the slope in DA, $\frac{b1-b2}{b2} < \frac{9d1}{2\alpha}$.

3.5.4 Welfare Change by Virtual Bid with Graph

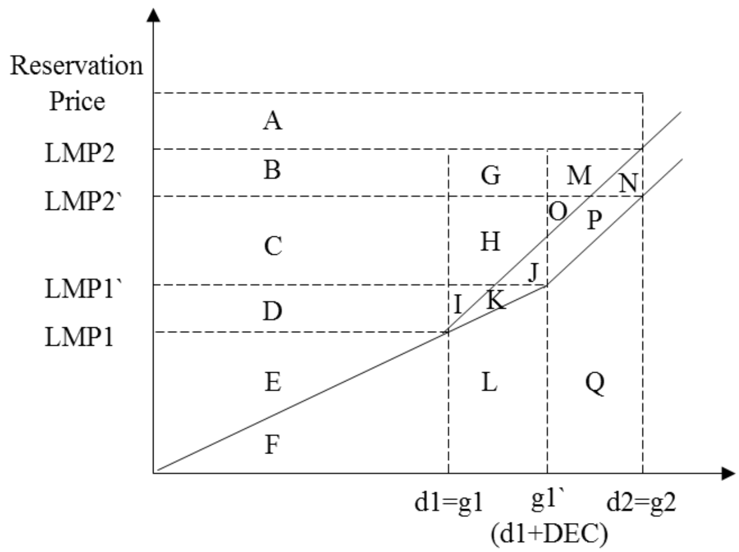

Figure 3.13 Welfare calculation when RT demand is greater than DA demand with a DEC

Figure 3.13 is the case when DA demand is lower than RT demand with a DEC making the committed generation in DA greater than DA demand and less than RT demand.

The consumer welfare change is a sum of Δ DA Payment by the consumer, and Δ RT balance payment by the consumer. The DA payment by the consumer without DEC is $\mathrm{d1}\cdot \mathrm{P1}(\mathrm{d1})$, the area of E+F and with DEC is $\mathrm{d1}\cdot \mathrm{P1}(\mathrm{d1}-\mathrm{V}^{*})$, or the area D+E+F with DEC. Thus, Δ DA Payment by the consumer, $-\mathrm{d1}\cdot(\mathrm{P1}(\mathrm{d1}-\mathrm{V}^{*})-\mathrm{P1}(\mathrm{d1})$, is $-$D. The RT balancing payment by the consumer without DEC is $\mathrm{E}[-\Delta\cdot \mathrm{P2}(\mathrm{d1}+\Delta)]$ or the area of –(G+H+I+J+K+L+M+N+O+P+Q) and with DEC is $\mathrm{E}[-\Delta\cdot \mathrm{P2}'(\mathrm{d1}+\Delta)]$, or the area of –(H+I+J+K+L+O+P+Q). Thus, Δ RT balance payment by the consumer, $\mathrm{E}[-\Delta\cdot \mathrm{P2}'(\mathrm{d1}+\Delta)]-\mathrm{E}[-\Delta\cdot \mathrm{P2}(\mathrm{d1}+\Delta)]$ is G+M+N. The overall welfare change on consumer due to virtual transaction is –D+G+M+N.

The consumer welfare change is a sum of Δ DA Payment to the producer, Δ RT Balancing Payment to the producer, and Δ Generation cost. The DA Payment to the producer without DEC is $d1 \cdot P1(d1)$ or the area of E+F and with DEC is $(d1 - V^*) \cdot P1(d1 - V^*)$ or the area of D+E+F+I+K+L. Hence, Δ DA Payment to the producer is $(d1 - V^*) \cdot P1(d1 - V^*)$ $- d1 \cdot P1(d1)$, D+I+K+L. The RT Balancing Payment to the producer without DEC is $E[\Delta \cdot P2(d1 + \Delta)$, or the area of G+H+I+J+K+L+M+N+O+P+Q and with DEC is $E[(\Delta + V^*) \cdot P2'(d1 + \Delta)]$, or the area of O+P+Q. Thus, Δ RT Balancing Payment to the producer is $-$ (G+H+I+J+K+L+M+N). The Generation cost without DEC is the area under the inverse supply curve up to d2, $-\frac{1}{2\alpha}\left[\int_{\mu-\alpha}^{0}\left(\int_{0}^{d1} P1(g)dg\right)d\Delta + \int_{0}^{\mu+\alpha}\left\{\int_{0}^{d1} P1(g)\,dg + \int_{d1}^{d1+\Delta} P2(g)\,dg\right\}d\Delta\right]$, or $-$(F+J+K+L+N+P+Q) and with DEC is $-\frac{1}{2\alpha}\left[\int_{\mu-\alpha}^{-V^*}\left(\int_{0}^{d1+\Delta} P1(g)dg\right)d\Delta + \int_{-V^*}^{\mu+\alpha}\left\{\int_{0}^{d1-V^*} P1(g)\,dg + \int_{d1-V^*}^{d1+\Delta} P2'(g)\,dg\right\}d\Delta\right]$, or $-$(F+L+Q). Thus, Δ Generation cost is J+K+N+P. The overall welfare change on producer due to the virtual transaction is D-G-H+K-M+P.

The financial trader surplus is $E[V(LMP1' - LMP2')] = V(P1(d1 - V) - E[P2'(d1 + \Delta)])$, which is H+J. With DEC bid, V is negative and the virtual demand bid pays in the DA market and have balancing payments in the RT market. The DA payment to the financial trader is $V(P1(d1 - V) = V \cdot LMP1'$, the area of I+K+L. RT payment by the financial trader is $E[P2'(d1 + \Delta)]) = V \cdot LMP2'$, the area of H+I+J+K+L. The total financial trader surplus is $E[V(LMP1' - LMP2')]$, the area of H+J, or $V(P1(d1 - V) - E[P2'(d1 + \Delta)])$, the area of –(I+K+L)+(H+I+J+K+L) which is positive.

3.6 Summary of Expected Welfare Changes by Virtual Bid

Table 3.1 Welfare changes with expected demand deviation and optimal bidding

Economic Welfare	$E[\Delta]=0$ DEC	$E[\Delta]>\delta^*$ DEC	$E[\Delta]<\delta^*$ INC
Virtual	Positive	Positive	Positive
Consumer	Decrease	Decrease	Increase
Producer	Increase	Increase	Decrease

The mathematical analysis helps us to understand the expected change of the welfare of market participants due to the introduction of optimal virtual bids. If the expectation of demand deviations is zero, a DEC is preferred. With a DEC, the expected consumer welfare is generally decreased by the inflated DA payment by the consumer due to the increased DA price, while the balancing payment by the consumer in RT is decreased. The expected producer welfare is generally increased by the inflated DA payment to the producer due to the increased DA price and reduced generation cost while the balance payment to the producer in RT is decreased.

Certain extreme sets of parameters for DA and RT slope relative to the DA demand and the range of uncertainty, however, may change the qualitative welfare impact of market participants due to the different functional form. When the range of uncertainty is 10% of DA demand, the consumer welfare decreases when the slope in RT is not greater than 37 times steeper than the slope in DA and the producer welfare increases when the slope in RT is not greater than 45 times steeper than the slope in DA.

When the expected demand deviation is more negative than the critical threshold derived above, an INC is preferred. With an INC, the expected consumer welfare is increased by the decreased DA payment and RT balance payment by the consumer due to the decreased DA price and the increased balancing payment to the consumer in the RT market. The expected producer welfare is decreased by the decreased DA payment to the producer due to the decreased DA price and increased balance payment by the producer in RT.

With sufficiently large negative expected demand deviations, INCs yield expected profits, while with relatively minor negative or zero expected demand deviation, DECs are profitable. The magnitude of the expected demand deviation relative to parameters such as the range of uncertainty, α, the slope of supply function in DA and RT markets qualify the characterization of the qualitative optimal bidding strategies.

CHAPTER 4. ELECTRICITY NETWORK AND MULTI-SETTLEMENT WHOLESALE ELECTRICITY MARKET

The electricity network is a set of lines connecting different locations. The intersections of these power lines are called network buses. At each network bus, there might be generators and loads. Positive and negative net power inflow and outflow at different buses cause power flows throughout the network. In analyzing electricity networks, we adopt the standard DC approximation assumption. We assume zero transmission losses. This is a standard assumption adopted in the academic literature and industry practice in order to simplify computational outcomes.

4.1 Loop Flow in the Electricity Network

To understand the nature of the electricity system, it is important to introduce some basic physical properties of the electricity network. When power moves between buses of the electricity network, they follow an inverse reactance rule and loop flow. The rule of inverse reactance is that the share of power that flows along each path is inversely proportional to the relative reactance of such a path (Kristiansen 2004). The lower the reactance of a line, the larger share of power flows on that path.

In addition, these laws explain flows in cases where there are "loops" in the network – that is a minimum of three buses that are all interconnected directly. Thus, if the buses are labeled 1, 2, and 3, there would be lines directly connecting 1 to 2, 2 to 3, and 1 to 3. This defines a loop of connections from 1 to 2 to 3 and back to 1. Loop flow addressed by the Kirchhoff's laws reflects the fact that power flow takes the "path of the least reactance" rather than taking the shortest delivery path. That is, if there is an injection of power at bus 1 and a withdrawal at bus 3, the flow will occur not only on the line from 1 to 3 but also on 1 to 2 and 2 to 3, with the amount of power flowing on each line related to the reactance of the line. Thus, while the desired flow may be from 1 to 3, the actual flow throughout the network obeys Kirchhoff's laws. Of course, loops cannot occur in a single bus or two bus network – three is a minimum number of buses before a loop can occur. Nevertheless, noting that simply having three buses does not necessarily create a loop. If the network topology is such that two of the buses are not directly connected, then the network is "radial" and no loops occur. Networks that include loops are the focus of this study.

4.2 Illustrative Example of Loop Flow

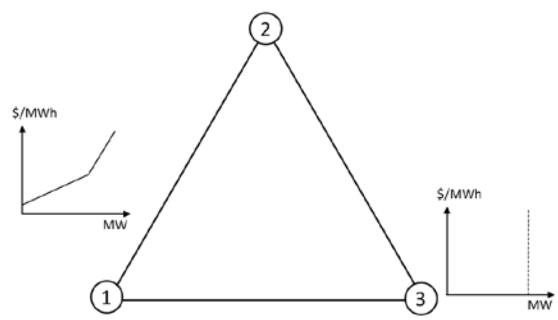

Figure 4.1 Generation unit and load

Assuming that we have a network composed of three buses and we have a generation unit (GenCo) at bus 1 (injection) and load (100 MW) at bus 3 (withdrawal). These three buses are connected by three lines are line 1-2, line 1-3 and line 2-3. Suppose that all the lines have the same reactance.

In order to assess the power flow caused by the injection at a specific bus, we need to designate a reference bus in the network. The choice of reference bus is arbitrary and, we take bus 3 for this illustrative example.

Because of loop flow, power flows on every path from bus 1 to bus 3. Due to the power injection in bus 1, the power flow from bus 1 to bus 3 moves along the two possible paths – line 1-3 or the other path, over line 1-2 and line 2-3. Since all the lines have the same reactance, the reactance of a path depends on the number of lines in the path. So, the reactance associated with the direct path of line 1-3 is half of the reactance of the indirect paths that are line 1-2 and line 2-3.

4.2.1 GenCo and Load with Unlimited Line Capacity

Assume that there is no limit on available transmission capacity on any of the three lines. Following the rule of inverse reactance, we can calculate the share of power injected in bus 1 that flows along each of the two possible paths. The share of power that flows along each path is inversely proportional to the relative reactance of such a path. This share is called the Power Transmission Distribution Factor (PTDF) and it can be calculated as

$PTDF_{1-3} = (R_{1-2}+ R_{2-3}) / (R_{1-2}+ R_{2-3}+ R_{1-3}) = 2R/3R= 2/3$

$PTDF_{1-2,2-3} = (R_{1-3}) / (R_{1-2}+ R_{2-3}+ R_{1-3}) = 1R/3R= 1/3$

Hence, we can conclude that one-third of the power injected at bus 1 will move along line 1-2 and 2-3 and two-thirds of that power will move along line 1-3.

Since we know the generation unit at bus 1 will produce energy to meet 100 MW of load (demand) at bus 3, we can calculate the amount of power flows in each of the two paths. Power flow along the line 1-2 and 2-3 are 1/3 of 100 MW, 33.3 MW and along the line, 1-3 is 2/3 of 100MW, 66.7 MW.

4.2.2 GenCo and Load with Limited Line Capacity under Loop Flow

We can adjust the assumption of unlimited line capacity by introducing a line capacity constraint of 50 MW on line 1-3. The introduction of such a constraint has an impact on the maximum amount of power that can be moved from bus 1 to bus 2, even if the capacity of the other lines are unlimited.

When there is a single generation unit in the network and the load is located on a different bus than the generation unit, having a congested line due to the line capacity constraint can limit the generated electricity. In this example, even though the generation unit at bus 1 can generate 100 MW to meet the load of 100 MW at bus 3, the loop flow does not allow them to deliver it. The net injection of 100 MW at bus 1 causes the power flow of 66.7 MW that must move along the line 1-3. However, the capacity constraint of 50 MW does not allow them to move along.

4.2.3 Two GenCos and Load with Unlimited Line Capacity and Symmetric Line Reactance

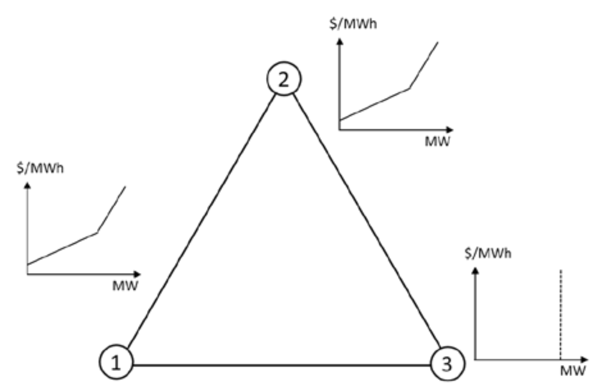

Figure 4.2 Two GenCos and load

In order to further determine the implications of the loop flow and PTDF, we consider a three-bus network with two buses with generation units (bus 1 and bus 2) having identical attributes and one bus with a load of 100 MW (bus 3). This means that bus 1 and bus 2 are net injection buses and bus 3 is net withdrawal bus of 100 MW. We assume that the three transmission lines have equal reactance and unlimited transmission capacity for the time being and these conditions will be relaxed later. Both generators have the same linear cost function and the magnitude of the slope is one. It costs $100 to generate 100 MW. Hence, in the unlimited line capacity case, each generates half of the load of 100 MW, which is 50 MW.

To identify the implications of the loop flow, first, we consider the power injection of 50 MW in bus 1. As addressed, due to loop flow, the power flow from bus 1 to bus 3 can move along two possible paths: the direct path, (1-3), or the indirect path (1-2 and 2-3). The PTDF of line 1-3 is 2/3 and PTDFs of line 1-2 and line 2-3 are 1/3 due to the identical line reactance (symmetric PTDF). The power flow on line 1-3 is 2/3 of 50 MW, 33.3 MW and the power flow in line 1-2 and 2-3 is 1/3 of 50 MW, 16.6 MW.

Second, we consider the power injection of 50 MW at bus 2. Two possible paths for power flow from bus 2 to bus 3 are line 2-3 and line 1-2, line 1-3. Following the demonstrated computation in the case of injection at bus 1, PTDF of line 2-3 is 2/3, PTDF of line 1-2 is -1/3 and PTDF of line 1-3 are 1/3. The direction of power flow by injection in specific lines refers to the reference bus (bus 3).

$PTDF_{2\text{-}3} = (R_{1\text{-}2} + R_{1\text{-}3}) / (R_{1\text{-}2} + R_{2\text{-}3} + R_{1\text{-}3}) = 2R/3R = 2/3$

$PTDF_{1\text{-}2} = -(R_{2\text{-}3}) / (R_{1\text{-}2} + R_{2\text{-}3} + R_{1\text{-}3}) = -1R/3R = -1/3$

$PTDF_{1\text{-}3} = (R_{2\text{-}3}) / (R_{1\text{-}2} + R_{2\text{-}3} + R_{1\text{-}3}) = 1R/3R = 1/3$

The power flow on line 2-3 is 2/3 of 50 MW, 33.3 MW, the power flow in line 1-2 is 1/3 of 50 MW in counter-flow direction, -16.6 MW and the power flow in line 1-3 is 1/3 of 50 MW, 16.6 MW.

We can determine the overall power flow over the transmission lines in the network by adding the calculated share of flows by injections at each bus. With injections of 50 MW in both bus 1 and bus 2, the overall power flow in line 1-2 is 16.6-16.6, 0 MW, in line 1-3 is 33.3+16.6, 50 MW and in line 2-3 is 16.6+33.3, 50 MW.

4.2.4 Two GenCos and Load with Unlimited Line Capacity and Asymmetric Line Reactance

To identify the influence of the line reactance on the power flow of the network, we can change the topology of line reactance. Instead of having identical reactance, assume line 1-3 has a reactance of R, which is half of the reactance of lines 1-2 and 2-3, or 2R. For the injection at bus 1, the calculation of PTDFs is

$PTDF_{1-3} = (R_{1-2}+ R_{2-3}) / (R_{1-2}+ R_{2-3}+ R_{1-3}) = 4R/5R= 4/5$

$PTDF_{1-2,2-3} = (R_{1-3}) / (R_{1-2}+ R_{2-3}+ R_{1-3}) = 1R/5R= 1/5$

A 50 MW injection at bus 1 results in power flow on line 1-3 of 4/5 of 50 MW, or 40 MW, and the power flow on line 1-2 and 2-3 is 1/5 of 50 MW or 10 MW.

For the injection at bus 2, the calculation of PTDFs is

$PTDF_{2-3} = (R_{1-2}+ R_{1-3}) / (R_{1-2}+ R_{2-3}+ R_{1-3}) = 3R/5R= 3/5$

$PTDF_{1-2} = -(R_{2-3}) / (R_{1-2}+ R_{2-3}+ R_{1-3}) = -2R/5R= -2/5$

$PTDF_{1-3} = (R_{2-3}) / (R_{1-2}+ R_{2-3}+ R_{1-3}) = 2R/5R= 2/5$

A 50 MW injection at bus 2 creates power flow on line 2-3 is 3/5 of 50 MW, 30 MW, the power flow on line 1-2 is 2/5 of 50 MW in counter-flow direction, or -20 MW, and the power flow in line 1-3 is 2/5 of 50 MW, or 20 MW.

In this asymmetric PTDF case, the injections of 50 MW at both bus 1 and bus 2 result in an overall power flow on line 1-2 of 10 minus 20, or -10 MW, on line 1-3 the flow is 40 plus 20, or 60 MW, and on line 2-3 it is 10 plus 30, or 40 MW.

4.2.5 Two GenCos and Load with Limited Line Capacity and Symmetric Line Reactance

The introduction of the line capacity constraint in the network with two generators has an impact on the decision of MW of generation. We assume that the capacity of line 1-3 is 45MW, 5 MW less than the power flow in the unlimited capacity case. In the symmetric PTDF case with identical line reactance, the power flow on line 1-3 by the injection at bus 1 is 2/3 of generation in bus 1 and by the injection at bus 2 is 1/3 of generation in bus 2, while the sum of generations must meet the load of 100 MW. Solving these equations, the generation at bus 1 is 35 MW and generation at bus 2 is 65 MW. Under the symmetric reactance case, due to the line capacity constraint:

Power flow on line 1-3 = Generation at bus 1 x 2/3 + Generation at bus 2 x 1/3 < 45 MW

Load at bus 3 = Generation at bus 1 + Generation at bus 2 = 100 MW

4.2.6 Two GenCos and Load with Limited Line Capacity and Asymmetric Line Reactance

Assume asymmetric line reactance with the line 1-3 having half of the reactance of lines 1-2 and 2-3. This causes a different set of generation decisions, although the other components in the network (line capacity, generation units, and load) are identical to the symmetric case.

Assume a capacity limit on line 1-3 is 55 MW in the asymmetric case, 5 MW less than its MW of power flow in the unlimited capacity case of 60 MW. The power flow on line 1-3 due to the injection at bus 1 is 4/5 of generation at bus 1 and due to the injection at bus 2 is 2/5 of generation at bus 2 while the sum of generations must meet the load of 100 MW. Solving these equations, the generation at bus 1 is 37.5 MW and generation at bus 2 is 62.5 MW.

Under the asymmetric reactance case:

Power flow on line 1-3 = Generation at bus 1 x 4/5 + Generation at bus 2 x 2/5 < 55 MW

Load at bus 3 = Generation at bus 1 + Generation at bus 2 = 100 MW

4.3 Bilevel Programming for Numerical Welfare Analysis

The problem of identifying the optimal bidding strategy for the virtual trader over a network-constrained two-settlement market can be numerically analyzed by bilevel programming. The proposed bilevel programming model contains an upper-level problem and a lower-level one that constrains the upper-level problem. The upper-level problem characterizes the profit maximization of the strategic virtual trader whose net revenues depend on the market clearing prices obtained in the lower-level problem. The lower-level problems represent the clearing of DA and RT markets with the target of maximizing expected social welfare subject to the producer, consumer, and electricity network constraints following DCOPF. The lower-level problems are replaced by its Karush–Kuhn–Tucker (KKT) conditions to convert the bilevel problem into a non-convex single-level optimization problem.

The multi-bus model, in contrast to the single bus model, generally has many suppliers and consumers located across the electricity network. The system operators manage the lower level electricity market consistently with the solution to an optimization model that focuses on delivering electricity to load at least cost and that also considers both economic and technical factors inherent to the system. The objective function is an expected social welfare maximization problem with the goal to maximize the surplus for the market participants such as consumers and producers. This is done by running an optimal power flow model constrained by different technical parameters such as generation capacities, transmission capabilities, and loads.

4.3.1 Upper-Level Problem: Financial Trader

The purpose of the upper-level problem is to find the optimal virtual product (INCs/DECs) bidding strategy for the financial trader in the network. This is done by maximizing the expected profit of the virtual bidding. The optimal virtual bidding problem is complicated by the fact that the calculation of the expected profit depends on the DA and RT market clearing prices at all buses. The clearing prices are determined by the generation dispatch of the system operator, for both the DA and RT markets. Note that the realized demand deviations are not known to the market operator or virtual trader at the time when the virtual trader must issue her/his bid in the DA market. Therefore, the optimal bidding strategy has to take into account the demand uncertainty in RT market. In the model presented here, this uncertainty is integrated into the model by multiple scenarios of lower-level problems indexed by ‘s’, and with associated probabilities Prob(s). Each scenario represents a realization of the demand deviation, which following a uniform distribution with exogenous probabilities of scenarios. That is,

$$\max \quad \sum_s \sum_b \mathrm{Prob}(s) \cdot V_b(\mathrm{LMP1}_b - \mathrm{LMP2}_{bs})$$

where V_b is the INC/DEC bid at bus b (positive V_b corresponds to an INC and negative V_b corresponds to a DEC), $\mathrm{LMP1}_b$ is the DA price at bus b, $\mathrm{LMP2}_{bs}$ is the RT price at bus b in scenario s and $\mathrm{Prob}(s)$ is the probability of the scenario s. The $\mathrm{LMP1}_b$ and $\mathrm{LMP2}_{bs}$, determined by lower-level DA and RT market problems, will be explained further in the lower level problem section below.

4.3.2 Lower-Level Problem: Day-Ahead Market

The upper-level and lower-level problems are interrelated since the upper-level problem of the virtual trader determines the optimal volume of virtual bids submitted to the DA market based on expected DA and RT prices, while the solution to the lower-level problems of the system operator determines the DA and RT prices have a direct influence on the virtual trader profit. The strategic virtual bids (V_b) are the upper-level decision variables treated as parameters in the lower-level problem. The virtual bids may alter the commitment of generation units and the MW of generation. The virtual demand bids, or DECs, are represented by $V<0$ and the virtual supply offers, or INCs, are represented by $V>0$ in the stylized DA market model.

The DA market in the lower level problem is to determine the market clearing pattern of generation and prices, which is based on economic social welfare maximization and optimal power flow formulations incorporating the INCs and DECs represented by V_b. That is,

$$\max \quad \sum_b RP_b \cdot d1_b - \sum_b \int_0^{g1_b} \left(-\frac{a1_b}{b1_b} + \frac{g}{b1_b}\right) dg$$

where RP_b is the consumer reservation price at bus b, $d1_b$ is consumer demand at bus b, $g1_b$ is the MW of cleared generation at bus b, $a1_b$ is a parameter of the DA supply function at bus b and $b1_b$ is a slope of the DA supply function at bus b. The consumer economic welfare summed across all nodes is $\sum_b RP_b \cdot d1_b$ and $\sum_b \int_0^{g1_b} -\frac{a1_b}{b1_b} + \frac{g}{b1_b} dg$ is generation cost (area under inverse supply function at node b) summed across all nodes. As RP_b and $d1_b$ are given in this problem, the system operator selects optimal generation level ($g1_b$) to maximize social welfare under the technical constraints.

The power balance constraint requires that total generation meets total demand in the network taking into account the virtual bids,

$$\sum_b (g1_b - (d1_b - V_b)) = 0, \quad \lambda 1 \geq 0$$

The Lagrange multipliers for these constraints are denoted by $\lambda 1$. Power flow limits at each transmission line are as follows:

$$\sum_b PTDF_{ijb} \, (g1_b - (d1_b - V_b)) \leq F_{ij}^{max}, \quad \pi 1_{ij}^{+} \geq 0, \qquad \forall_{ij}$$

$$-\sum_b PTDF_{ijb} \, (g1_b - (d1_b - V_b)) \leq F_{ij}^{max}, \quad \pi 1_{ij}^{-} \geq 0, \qquad \forall_{ij}$$

where F_{ij}^{max} is the transmission line capacity and $\pi 1_{ij}^{+}, \pi 1_{ij}^{-}$ are Lagrange multipliers. $\sum_b PTDF_{ijb} \, (g1_b - (d1_b - V_b))$ is the power flow on path ij reflecting all injections and withdrawals in the network, which must be between F_{ij}^{max} and $-F_{ij}^{max}$. $PTDF_{ijb}$ is incremental power flow on the line from i to j with respect to power injection at bus b and withdrawals at the reference bus integrating the physical laws of loop flow. All of the generation quantities must be non-negative values ($g1_b \geq 0$).

The FOCs w.r.t. $g1_b$ for the above DA PTDF problem are the following:

$$-\frac{a1_b}{b1_b} + \frac{g1_b}{b1_b} - \lambda 1 + \sum_i \sum_j \pi 1_{ij}^{+} PTDF_{ijb} - \sum_i \sum_j \pi 1_{ij}^{-} PTDF_{ijb} \geq 0 \quad \forall_b$$

With rearranged terms, this becomes $-\frac{a1_b}{b1_b} + \frac{g1_b}{b1_b} = \lambda 1 - \sum_i \sum_j PTDF_{ijb} \left(\pi 1_{ij}^{+} - \pi 1_{ij}^{-}\right)$.

Interpreting this economically, the incremental marginal cost of generation equals the definition of marginal value to consumers (price) at each node b. These marginal values to consumers define the locational marginal prices in the DA market ($LMP1_b$):

$$LMP1_b = \lambda 1 - \sum_i \sum_j PTDF_{ijb} \left(\pi 1_{ij}^{+} - \pi 1_{ij}^{-}\right)$$

where $\lambda 1$, $\pi 1_{ij}^{+}$ and $\pi 1_{ij}^{-}$ are Lagrangian multipliers. LMP represents the cost to the system of a unit increase in the load at the bus. In the absence of line capacity constraints and losses, all locational prices will be equal across all nodes b. With limiting transmission capacity constraints, the multipliers, $\pi 1_{ij}^{+}$ and $\pi 1_{ij}^{-}$, will adjust the optimal solutions to the problem. At changes in the constraints such as the available capacity, the multipliers would change the equilibrium solution, which leads to a price change. If the change does not reach a constraint limit, the multiplier will be zero, and there will be no change in the price. The energy price is $\lambda 1$ and $-\sum_i \sum_j PTDF_{ijb} \left(\pi 1_{ij}^{+} - \pi 1_{ij}^{-}\right)$ is congestion price.

4.3.3 Lower-Level Problem: Real-Time Market

The Real-Time (RT) market in the lower-level problem is also an economic social welfare maximization problem with optimal power flow. However, it is different from the DA market problem. The generation dispatch decision in the RT market is based on committed generation units determined in the DA market. Particularly when there is a need to dispatch additional MW of energy in the RT market due to the positive deviation of the realized demand, the system operator has to have fast ramping high-cost peaker units to meet demand in a short time that are following the RT supply curve. The RT market does not consider virtual bids in the optimization problem since the system operator makes dispatch decisions based on physical demands. While the demands in the DA market are fixed at the planning stage, the demands in the RT market are uncertain with known distribution. As discussed above, the probabilistic nature of the RT demands is implemented via a set of scenarios that consider expected possible realizations of the uncertain demand, which occur with exogenous probabilities.

The economic social welfare maximizing objective function in the RT market is a following:

$$\max \quad \sum_s \sum_b \mathrm{Prob}(s) \cdot \mathrm{RP}_b \cdot \mathrm{d1}_b$$

$$-\sum_s \sum_b \mathrm{Prob}(s) \cdot \left\{ \int_0^{\min(g1_b, g2_{bs})} \left(-\frac{a1_b}{b1_b} + \frac{g}{b1_b} \right) dg \ + \int_{g1_b}^{\max(g1_b, g2_{bs})} \left(-\frac{a2_b}{b2_b} + \frac{g}{b2_b} \right) dg \right\}$$

where $g2_b$ is the MW of cleared generation at bus b in scenario ‘s’, $a2_b$ is a parameter of RT supply function at bus b and $b2_b$ is the slope of RT supply function at bus b. In this case, $\sum_s \sum_b \mathrm{Prob}(s) \cdot \left\{ \int_0^{\min(g1_b, g2_{bs})} \left(-\frac{a1_b}{b1_b} + \frac{g}{b1_b} \right) dg \ + \int_{g1_b}^{\max(g1_b, g2_{bs})} \left(-\frac{a2_b}{b2_b} + \frac{g}{b2_b} \right) dg \right\}$ represents the total generation cost in scenario ‘s’ according to the realized demands in RT market. For negative deviations of generation, the second integral vanishes because the lower and upper limits of integration are the same. For positive deviations of generation, the marginal cost is set by the RT supply curve.

The power balance constraint, which $\lambda 2_s$ is the Lagrange multiplier for the following constraint:

$$\sum_b (g2_{bs} - d2_{bs}) = 0, \qquad \lambda2_s \geq 0, \qquad \forall_s$$

where generation in every state at every node must be non-negative ($g2_{bs} \geq 0$). In each scenario, the sum of generations in the network ($\sum_b g2_{bs}$) must meet the sum of deviations in realized demands in the network ($\sum_b g2_{bs} = \sum_b \Delta_{bs}$).

Demand in the RT market in scenario 's', $d2_{bs}$, is the realization of the uniform distribution of demand deviation plus demand in DA market ($d1_b$+Δ_{bs}). The known distributions of the demand uncertainty (Δ_{bs}) are assumed to be uniform for purposes of analysis within the range of ($\mu_b - a_b$, $\mu_b + a_b$), and deviations are assumed to be independent across nodes.

Power flow limits at each transmission lines are a follows:

$$\sum_b PTDF_{ijb} (g2_{bs} - d2_{bs}) \leq F_{ij}^{max} , \qquad \pi2^+_{ijs} \geq 0, \qquad \forall_{ijs}$$

$$-\sum_b PTDF_{ijb} (g2_{bs} - d2_{bs}) \leq F_{ij}^{max} , \qquad \pi2^-_{ijs} \geq 0, \qquad \forall_{ijs}$$

where $\pi2^+_{ijs}$ and $\pi2^-_{ijs}$ are Lagrange multipliers.

The FOC w.r.t. $g2_{bs}$ for the above RT PTDF problem is a follows:

$$\max(-\frac{a1_b}{b1_b} + \frac{g2_{bs}}{b1_b}, -\frac{a2_b}{b2_b} + \frac{g2_{bs}}{b2_b}) - \lambda2_s + \sum_i \sum_j \pi2^+_{ijs} PTDF_{ijb} - \sum_i \sum_j \pi^-_{ijs} PTDF_{ijb} \geq 0 \qquad \forall_{bs}$$

which can be rearranged in the case when $g2_{bs} > 0$ as:

$$\max(-\frac{a1_b}{b1_b} + \frac{g2_{bs}}{b1_b}, -\frac{a2_b}{b2_b} + \frac{g2_{bs}}{b2_b}) = \lambda2_s - \sum_i \sum_j PTDF_{ijb} \left(\pi2^+_{ijs} - \pi2^-_{ijs}\right).$$

This yields that the incremental marginal cost of generation equals the marginal value of power to consumers (price). The cleared locational marginal prices in the RT market in scenario s ($LMP2_{bs}$) is stated as follows:

$$LMP2_{bs} = \lambda2_s - \sum_i \sum_j PTDF_{ijb} \left(\pi2^+_{ijs} - \pi2^-_{ijs}\right)$$

where $\lambda2_s$, $\pi2^+_{ijs}$ and $\pi2^-_{ijs}$ are Lagrangian multipliers. The energy price is $\lambda2_s$ and $-\sum_i \sum_j PTDF_{ijb} \left(\pi2^+_{ijs} - \pi2^-_{ijs}\right)$ is the congestion price.

4.3.4 Bilevel Formulation and KKT Reformulation

The bilevel formulation for the stylized two-settlement electricity market with multi-bus network integrating loop flow is stated as following:

Upper-level: Virtual trader

$$\max \quad \sum_s \sum_b \mathrm{Prob}(s) \cdot V_b(\mathrm{LMP1}_b - \mathrm{LMP2}_{bs}) \qquad (4.1a)$$

s.t.

$$\mathrm{LMP1}_b = \lambda 1 - \sum_i \sum_j \mathrm{PTDF}_{ijb} \left(\pi 1_{ij}^{+} - \pi 1_{ij}^{-}\right) \qquad \forall_b$$

$$\mathrm{LMP2}_{bs} = \lambda 2_s - \sum_i \sum_j \mathrm{PTDF}_{ijb} \left(\pi 2_{ijs}^{+} - \pi 2_{ijs}^{-}\right) \qquad \forall_{bs}$$

Lower-level: Day-Ahead market

$$\max \sum_b \mathrm{RP}_b \cdot d1_b - \sum_b \int_0^{g1_b} -\frac{a1_b}{b1_b} + \frac{g}{b1_b} dg \qquad (4.2a)$$

s.t

$$\sum_b (g1_b - (d1_b - V_b)) \geq 0 \qquad \lambda 1 \geq 0 \qquad (4.2b)$$

$$\sum_b \mathrm{PTDF}_{ijb} \, (g1_b - (d1_b - V_b)) \leq F_{ij}^{max} \qquad \pi 1_{ij}^{+} \geq 0 \qquad \forall_{ij} \qquad (4.2c)$$

$$-\sum_b \mathrm{PTDF}_{ijb} \, (g1_b - (d1_b - V_b)) \leq F_{ij}^{max} \qquad \pi 1_{ij}^{-} \geq 0 \qquad \forall_{ij} \qquad (4.2d)$$

Lower-level: Real-Time market

$$\max \sum_s \sum_b \mathrm{Prob}(s) \cdot \mathrm{RP}_b \cdot d1_b$$

$$-\sum_s \sum_b \mathrm{Prob}(s) \cdot \left(\int_0^{\min(g1_b, g2_{bs})} \left(-\frac{a1_b}{b1_b} + \frac{g}{b1_b}\right) dg \; + \int_{g1}^{\max(g1_b, g2_{bs})} \left(-\frac{a2_b}{b2_b} + \frac{g}{b2_b}\right) dg \right) \qquad (4.3a)$$

s.t

$$\sum_b (g2_{bs} - d2_{bs}) \geq 0 \qquad \lambda 2_s \geq 0 \qquad \forall_s \qquad (4.3b)$$

$$\sum_b \mathrm{PTDF}_{ijb} \, (g2_{bs} - d2_{bs}) \leq F_{ij}^{max} \qquad \pi 2_{ijs}^{+} \geq 0 \qquad \forall_{ijs} \qquad (4.3c)$$

$$-\sum_b \mathrm{PTDF}_{ijb} \, (g2_{bs} - d2_{bs}) \leq F_{ij}^{max} \qquad \pi 2_{ijs}^{-} \geq 0 \qquad \forall_{ijs} \qquad (4.3d)$$

The upper-level problem (4.1a) represents the objective function of the virtual trader maximization of trading profit. The lower-level problem (4.2a)–(4.2f) represents the DA market clearing process and (4.3a)–(4.3f) represents the RT market clearing process in order to maximize the social welfare as expressed by (4.2a) and (4.3a). Constraints on lower-level problems are system balance constraints (4.2b) and (4.3b), and power flow limits (4.2c)-(4.2d) and (4.3c)-(4.3d). The LMP of each bus is determined endogenously within the lower-level problem incorporating the volume of virtual bids from the upper-level problem.

The bilevel problem is non-convex even with the simple linear program formulation. In order to transform this bilevel problem into a single-level problem, the lower-level problem is replaced by its Karush–Kuhn–Tucker (KKT) conditions. The complementary slackness conditions between the constraints and their Lagrange multipliers makes the constraint set makes this single-level non-convex problem. For the purposes of this study, the stylized model represents a single hour market while the model can be extended to a multi-hour (one day) formulation**.** The KKT reformulation of the bilevel program has the computational advantage that is as a single level problem. The equations of KKT reformulation are presented as follows:

Upper-level: Virtual trader

max $\quad \sum_s \sum_b \mathrm{Prob}(s) \cdot V_b (LMP1_b - LMP2_{bs})$

$$LMP1_b = \lambda 1 - \sum_i \sum_j PTDF_{ijb} \left(\pi 1^{+}_{ij} - \pi 1^{-}_{ij}\right) \qquad \forall_b$$

$$LMP2_{bs} = \lambda 2_s - \sum_i \sum_j PTDF_{ijb} \left(\pi 2^{+}_{ijs} - \pi 2^{-}_{ijs}\right) \qquad \forall_{bs}$$

s.t

Lower-level: Day-Ahead market

$$-\frac{a1_b}{b1_b} + \frac{g1_b}{b1_b} - \lambda 1 + \pi 1^{+}_{ij} \sum_i \sum_j PTDF_{ijb} - \pi 1^{-}_{ij} \sum_i \sum_j PTDF_{ijb} = 0 \qquad \forall_b$$

$$-\sum_b (g1_b - (d1_b - V_b)) + S\_\lambda 1 = 0$$

$\sum_b PTDF_{ijb}\,(g1_b-(d1_b-V_b)) - F_{ij}^{max} + S\_\pi1_{ij}^{+} = 0 \qquad \forall_{ij}$

$-\sum_b PTDF_{ijb}\,(g1_b-(d1_b-V_b)) - F_{ij}^{max} + S\_\pi1_{ij}^{-} = 0 \qquad \forall_{ij}$

$0 \le S\_\lambda1 \perp \lambda1 \ge 0$

$0 \le S\_\pi1_{ij}^{+} \perp \pi1_{ij}^{+} \ge 0 \qquad \forall_{ij}$

$0 \le S\_\pi1_{ij}^{-} \perp \pi1_{ij}^{-} \ge 0 \qquad \forall_{ij}$

Lower-level: Real-Time market

$\max(-\frac{a1_b}{b1_b}+\frac{g2_{bs}}{b1_b},-\frac{a2_b}{b2_b}+\frac{g2_{bs}}{b2_b})$

$-\lambda2_s + \pi2_{ijs}^{+}\sum_i\sum_j PTDF_{ijb} - \pi_{ijs}^{-}\sum_i\sum_j PTDF_{ijb} = 0 \qquad \forall_{bs}$

$-\sum_b(g2_{bs}-d2_{bs}) + S\_\lambda2_s = 0 \qquad \forall_s$

$\sum_b PTDF_{ijb}\,(g2_{bs}-d2_{bs}) - F_{ij}^{max} + S\_\pi2_{ijs}^{+} = 0 \qquad \forall_{ijs}$

$-\sum_b PTDF_{ijb}\,(g2_{bs}-d2_{bs}) - F_{ij}^{max} + S\_\pi2_{ijs}^{-} = 0 \qquad \forall_{ijs}$

$0 \le S\_\lambda2_s \perp \lambda2_s \ge 0 \qquad \forall_s$

$0 \le S\_\pi2_{ijs}^{+} \perp \pi2_{ijs}^{+} \ge 0 \qquad \forall_{ijs}$

$0 \le S\_\pi2_{ijs}^{-} \perp \pi2_{ijs}^{-} \ge 0 \qquad \forall_{ijs}$

4.4 Welfare of Market Participants in Multi-Bus Model

The basic principles to calculate the welfare of market participants in the multi-bus model are identical to the ones in the single-bus model. The consumer welfare calculation is the negative of the consumer payment in Day-Ahead (DA) plus balancing payments for the consumer in the RT market plus consumer value. The producer welfare calculation is payment to the generator in the DA market plus net balancing payments to the producer minus generation cost. Social welfare is the sum of consumer and producer welfare. The financial trader welfare surplus calculation is net payments to the financial trader in the DA market plus net balancing payments to the financial trader in the RT market.

For the consumer surplus at bus b, the DA payment by the consumer is the cleared quantity of physical demand in DA ($d1_b$) times the market price, or $d1_b \cdot LMP1_b$. The RT balancing payment to the consumer is a financial settlement for the demand deviation in RT market from cleared demand in DA market. The calculation of RT net balancing payment to the consumer at bus b is the difference between DA demand and RT demand times RT price, $(d1_b - d2_b) \cdot LMP2_b$.

When the realized demand in RT is less than cleared demand in DA, the balance payment to the consumer is positive as the system operator pays back for the unused energy at the RT price. When the realized demand in the RT market is larger than DA demand, the balancing payment to the consumer is negative as the consumer must pay for the additional energy use at the RT price. The perceived value of electricity to consumers can be presented as a reservation price, RP, that always has a greater fixed value than the electricity price. Consumer value is the realized demand in the RT market times the reservation price, $d2_{bs} \cdot RP$. Overall, the consumer surplus calculation for bus b is $-d1_b \cdot LMP1_b + (d1_b - d2_b) \cdot LMP2_b + d2_b \cdot RP$.

Total consumer surplus is the sum of consumer surpluses at each bus across the network, or $\sum_b [-d1_b \cdot LMP1_b + (d1_b - d2_b) \cdot LMP2_b + d2_b \cdot RP]$.

For the producer surplus at bus b, the DA payment to the producer is the cleared quantity of physical generation times the market price in DA, or $g1_b \cdot LMP1_b$. The RT balancing payment to the producer at bus b is the difference between RT generation and DA generation times the RT price, $-(g1_b - g2_b) \cdot LMP2_b$. When realized generation in the RT market is greater than the cleared generation in the DA market, the balancing payment to the producer is positive as system operator pays for the additional generation at the RT price. When realized generation in the RT market is less than the cleared generation in the DA, the balancing payment to the producer is negative as the producer needs to return payments for electricity not supplied at the RT price. Generation cost at bus b is equivalent to the area under the supply curve to the realized generation in the RT market, or $\int_0^{\min(g1_b, g2_b)} P1(g)\, dg + \int_{g1_b}^{\max(g1_b, g2_b)} P2(g)\, dg$. Overall, the producer surplus calculation is $g1_b \cdot LMP1_b - (g1_b - g2_b) \cdot LMP2_b - \left(\int_0^{\min(g1_b, g2_b)} P1(g)\, dg + \int_{g1_b}^{\max(g1_b, g2_b)} P2(g)\, dg\right)$.

Producer Surplus in the network is the sum of producer surpluses at each bus

$$= \sum_b \left[g1_b \cdot LMP1_b - (g1_b - g2_b) \cdot LMP2_b - \left(\int_0^{\min(g1_b, g2_b)} P1(g)\, dg + \int_{g1_b}^{\max(g1_b, g2_b)} P2(g)\, dg\right)\right]$$

Based on the welfare formulations, the consumer welfare differences come from DA payments by the consumer and net RT balancing payments by the consumer. Producer welfare differences are from the DA payment to the producer, RT balancing payments to the producer and realized generation cost.

For the financial trader surplus, the DA payment to the financial trader is a cleared bid times the DA price or V·LMP1. RT payment by the financial trader is a spontaneous settlement at RT market price by the system operator. Thus, the financial trader bids an INC, the positive quantity of V in the model, in the hope that the DA price will be larger than the realized RT price. Analogously, the financial trader bids a DEC, a negative value of V in the model, in the hope that the DA price will be lower than the realized RT price. Overall, the financial trader surplus calculation at bus b is the expected profit earned at that bus, $V_b(LMP1'_b - LMP2'_{bs})$. The total financial trader surplus across the network is $\sum_b V_b(LMP1_b - LMP2_{bs})$.

4.5 Welfare of Market Participants in Multi-Bus Model: Uncongested Network

In the single bus model, the DA and RT market prices (LMP1 and LMP2) are determined by the characteristics of the generation unit and load represented by the supply function parameters and demand uncertainty distribution.

In an uncongested network, the system operator can dispatch the generators in merit order without transmission line constraints as the generated energy can be delivered to the load(s) in the network without limitations. Having demand uncertainty with a known distribution, the system operator can plan and dispatch generators accordingly in merit order regardless of their location. Hence, when the aggregated physical characteristics of the generator(s) and load(s) are identical, the market outcomes are identical regardless of the location of the generator(s) and load(s) as well as the type of PTDF.

When the aggregated physical characteristics of the generation unit(s) and load(s) in the single bus model and the uncongested multi-bus model are identical, the clearing prices are identical between the single bus and the uncongested multi-bus models for both the DA and RT markets. When the virtual bids in the single bus model and the sum of virtual bids in the multi-bus model are identical, the realized prices in DA and RT markets are identical.

$$\mathrm{LMP1} = \mathrm{LMP1}_1 = \mathrm{LMP1}_2 = \mathrm{LMP1}_3 \ldots = \mathrm{LMP1}_\mathrm{b}$$

$$\mathrm{LMP2} = \mathrm{LMP2}_1 = \mathrm{LMP2}_2 = \mathrm{LMP2}_3 \ldots = \mathrm{LMP2}_\mathrm{b}$$

If $\mathrm{V}^* = \sum_\mathrm{b} \mathrm{V}_\mathrm{b}^* = \mathrm{V}_1^* + \mathrm{V}_2^* + \mathrm{V}_3^* \ldots + \mathrm{V}_\mathrm{b}^*$

$$\mathrm{LMP1}' = \mathrm{LMP1}_1' = \mathrm{LMP1}_2' = \mathrm{LMP1}_3' \ldots = \mathrm{LMP1}_\mathrm{b}'$$

$$\mathrm{LMP2}' = \mathrm{LMP2}_1' = \mathrm{LMP2}_2' = \mathrm{LMP2}_3' \ldots = \mathrm{LMP2}_\mathrm{b}'$$

While the expected prices in DA and RT markets are identical in the single bus and the uncongested multi-bus models, the optimal bidding strategy must be identical. The overall optimal bidding volume in the multi-bus model is the same as the volume in the single bus model.

$$V^* = \sum_b V_b^*$$

The sum of optimal virtual bids in the network is identical to the single bus case and the identical optimal bidding strategy generates identical market outcomes that give identical welfare impact to the market participants.

CHAPTER 5. WELFARE ANALYSIS IN THE SIMPLIFIED NETWORKS

The fundamental goal of this study is to understand the impact of line congestion and loop flow on the changes in expected welfare of market participants due to the introduction of virtual transactions. We will use different types of networks to determine whether having line congestion and loop flow in the network amplify or dampen the expected welfare impacts determined in the absence of congestion and loop flow.

In the absence of active line capacity limits and without line losses, results of the optimal power flow model will result in a single price that clears markets throughout the network. That is, the laws of power flow that govern the movement of energy in the network do not affect the market outcomes.

Line congestion can alter the central dispatch decisions regarding power generation, the pattern of power flow in the network and as a consequence, the prices of electricity at the bus level. In this regard, having a congested line may reshape the optimal virtual bidding strategy relative to the uncongested case. The initial analysis will start with small, simple networks to assess the impacts of virtual transactions on agent welfare for uncongested and congested cases.

The next step in the analysis will augment the initial model to introduce loops in the network topology. In the presence of loops and congestion, the electricity flows in the network are governed by Kirchoff's Laws. When the associated loop flow exists, the shares of power flow along the lines are determined by the relative magnitudes of the line reactance in the electricity network. The shares are called the Power Transfer Distribution Factors (PTDFs) and the different combinations of line reactance result in different PTDFs. The configuration of line reactance and the existence of line congestion can change the impact of the inflow and outflow of energy and hence change the market results, including nodal prices, and the optimal bidding strategy for virtual market participants. In turn, this may have impacts on the expected welfare changes associated with the introduction of virtual bids.

In the stylized model, the virtual transaction may be bid at any bus in the network, reflecting the electricity market practice that virtual supply and virtual demand bids are permitted at most locations in the network. We assume a single financial trader unilaterally optimizes profits from virtual bids throughout the network. This is not a rare case in real electricity market operation where some areas of the network have very few virtual bidders. The most interesting range of expected demand deviation is near zero. It is the ideal case for system operator where forecasted DA demand and realized RT demand are identical, yet virtual trader still makes a profit in the stylized model. Based on data from PJM, this also appears to be a reasonable assumption.

This section focuses on estimating the expected differential welfare impact due to the introduction of virtual transactions in the case where there is congestion in the power network. In the interest of parsimony, the models addressed here have two buses (labeled 1 and 2) with a single line connecting them and three buses (labeled 1, 2 and 3) with all pairs of buses connected by single lines. In both cases, the line capacity is adjusted to generate different levels of congestion on the line from bus 1 to bus 2 to assess the expected welfare impact on market participants while all the other physical characteristics are fixed.

In the two settlement market, there is the possibility of congestion in the day ahead market, and there is also the possibility of congestion in the real-time market, given the realized real-time demand. Line capacity is varied across three cases: A) always uncongested, B) always congested, and C) occasionally congested. When the line capacity is unlimited, the network is always uncongested in DA and RT markets. When the line capacity is low, the network is always congested in DA and RT markets. When the line capacity is near the MW of power flow on that line without a virtual transaction, the line may be occasionally congested considering the demand uncertainty in RT. For example, the line can be uncongested in DA but congested or uncongested in RT depending on the realization of RT demand, or it may be congested in DA but congested or uncongested in RT depending on the realization of RT demand.

5.1 Two-Bus and Three-Bus Models

In this section, we introduce two- and three-bus models to identify the impact of the virtual transaction in the electricity network with congestion and loop flow. Two-bus model is the simplest form of a network having transmission line that can be congested due to line capacity constraint. Three-bus model is the simplest network to integrate loop flow in the network that can apply Kirchhoff's law.

5.1.1 Two-Bus Model Configuration

For simplicity, we begin with a two-bus model where each bus has a generator and load, and the buses are connected by a transmission line. While a single bus model obviously cannot exhibit congestion, a two-bus model with a line connecting buses could be congested. The goal of this section is to estimate the changes in expected welfare impact due to the introduction of the virtual transaction in the congested network relative to the uncongested network representing the single-bus case.

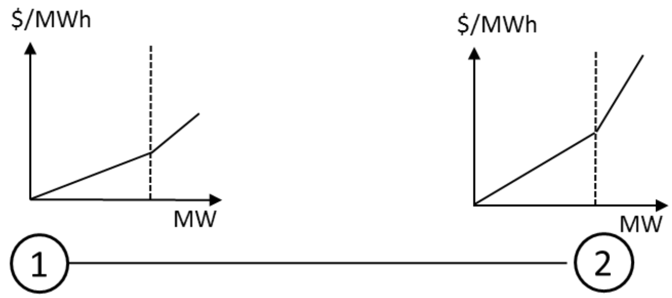

Figure 5.1 Two-bus network

This study uses a linear supply function that could be viewed as an abstraction of the true step supply functions of the generation units. The parameters of supply curves are chosen so that the generation supply curve at bus one has lower marginal cost relative to the generation supply curve at bus two. In the DA market, the cleared volume of the load at bus one and bus two are chosen to be identical in order to make them comparable. In RT market, the RT demand is the sum of the determined loads in the DA market and the realized demand deviation in RT. The distributions of RT demand are assumed to be uniform on the interval $(\mu_b - \alpha_b, \mu_b + \alpha_b)$ where the α_b represents the range of uncertainty at bus b and the μ_b represents the mean of the distribution at bus b. In this test case, we assume that the realizations of the demand deviations are independent of each other and μ_b. In line with typical deviations between DA and RT demand, the range of demand uncertainty is assumed to be $\pm$10% of the load. In this regard, the direction of power flow over the line is generally from bus 1 to bus 2 according to the merit order dispatch by the system operator.

Table 5.1 Two-bus model: Load data

Bus	DA Load (MW)	Distribution of load deviation*
1	200	[-10,10]
2	200	[-10,10]

* Uniform distribution represents the uncertainty of the load deviations.

Table 5.2 Two-bus model: Generation unit data

Bus	DA supply curve parameter (a1)	DA supply curve slope (b1)	RT supply curve slope (b2)
1	0	1	0.5
2	0	0.4	0.2

Table 5.3 Two-bus model: Line data

Type	Reactance	Loss	Capacity
Uncongested	0.1	0	95 MW
Congested	0.1	0	75 MW

* The power flow over the line from bus 1 to 2 without virtual transactions is 86 MW in DA.

5.1.2 Three-Bus Models Configuration

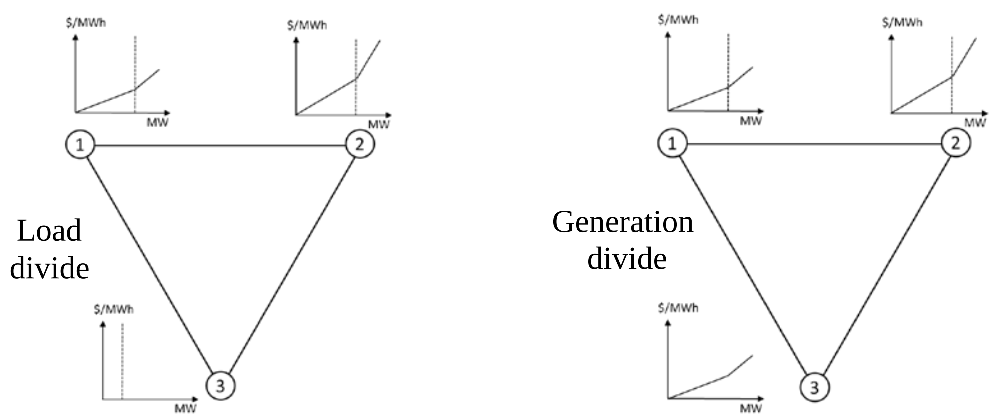

Figure 5.2 Three-bus networks

In aggregate, the three-bus network models have identical physical characters with two-bus network except having one more bus that is connected to the other buses with the load. We create two different types of three-bus network that maintain the same physical characteristics of the two-bus model.

The one is called load divide. The load located in bus one in the two-bus model is separated into bus one and bus three so the sum of the realized load in bus one and bus three in the three-bus model is identical to the realized load in bus one in the two-bus model.

The other one is called generation unit divide. The supply function in bus one in the two-bus model is separated into bus one and bus three so the vertical sum of the supply functions in bus one and bus three of the three-bus model is identical to the supply function in bus one of the two-bus model. The attributes of loads and generation units are identical, hence the power tends to flow from bus one to bus two.

The bold differences in power flow in the three-bus network are the power flows not directly via line 1-2 exclusively, but through parallel lines that are line 1-2, line 1-3 to line 3-2 due to loop flow in the closed network. The reactance of the lines is adjusted to have identical power flow in line 1-2, in two-bus and three-bus models. The heterogeneous line reactance is not an uncommon case in the electricity network.

Table 5.4 Three-bus model (Load divide): Load data

Bus	Load (MW)	Distribution of deviation*
1	200 × 0.8	[-10,10] × 0.8
2	200	[-10,10]
3	200 × 0.2	[-10,10] × 0.2

* Uniform distribution represents the uncertainty of the load deviations.

Table 5.5 Three-bus model (Load divide): Generation unit data

Bus	DA supply curve parameter (a1)	DA supply curve slope (b1)	RT supply curve slope (b2)
1	0	1	0.5
2	0	0.4	0.2
3	N/A	N/A	N/A

Table 5.6 Three-bus model (Load divide): Line data

From Bus	To Bus	Reactance	Loss	Capacity*
1	2	0.1	0	Uncongested: 95 MW Congested: 75 MW
1	3	0.0465	0	Unlimited
2	3	0.1	0	Unlimited

* The power flow over line 1-2 without virtual transaction is 86 MW

Table 5.7 Three-bus model (Generation unit divide): Load data

Bus	Load (MW)	Distribution of deviation*
1	200	[-10,10]
2	200	[-10,10]
3	N/A	N/A

* Uniform distribution represents the uncertainty of the load deviations.

Table 5.8 Three-bus model (Generation unit divide): Generation unit data

Bus	DA supply curve parameter (a1)	DA supply curve slope (b1)	RT supply curve slope (b2)
1	0	0.95	0.475
2	0	0.4	0.2
3	N/A	0.05	0.025

Table 5.9 Three-bus model (Generation unit divide): Line data

From Bus	To Bus	Reactance	Loss	Capacity*
1	2	0.005	0	Uncongested: 95 MW Congested: 75 MW
1	3	0.1	0	Unlimited
2	3	2	0	Unlimited

* The power flow over line 1-2 without virtual transaction is 86 MW

5.2 Welfare Impact of Virtual Transactions in an Electricity Network

The welfare impact of introducing optimal virtual bids in the electricity network will be identified with the different network cases that are uncongested, congested, and congested with loop flow. The uncongested case is imposing very large line capacity so there is no chance to have congestion in the network. The congested case is, in the two-bus model, the power flow (MW) binding to the line capacity. Congested with loop flow case is, in the thee-bus model, having congestion as in the congested case while there is a loop flow in the network. The terms generators, generation units, and producers are interchangeable. The term loads, load-serving entities, and demands are interchangeable.

5.2.1 Welfare Changes due to Introduction of Optimal Virtual Bidding in an Uncongested Network

The uncongested case can be regarded as a single-bus model. If the physical characteristics of loads and generation units are identical in the uncongested network, the optimal bidding strategy and its expected welfare impact on market participants are identical to the single-bus model. Throughout identical numerical results, we can assure that the physical settings of loads and generation units in two-bus and three-bus networks are identical.

As shown in the previous chapter, the uncongested network works as if it is a single bus network. In the uncongested network, the qualitative relationship between virtual transaction and the expected welfare impact is not homogeneous between market participants at the network level. The results of the expected welfare changes in the single-bus model are the same as the analytical model, as expected. When the expected demand deviation is zero, the optimal bidding strategy for the financial trader is a DEC and it makes negative expected welfare change in consumer and a positive expected welfare change in the producer. Having demand uncertainty, financial trader's expected welfare gain is always positive with optimal bidding strategy.

Table 5.10 Expected welfare impact due to the introduction of optimal virtual bidding: uncongested networks

	Consumer Welfare	Producer Welfare	Total Social Welfare	Virtual Trader Welfare
Uncongested two-bus	Decrease	Increase	Decrease	Increase
Uncongested three-bus	Decrease	Increase	Decrease	Increase

5.2.2 Welfare Changes due to Introduction of Optimal Virtual Bidding in a Congested Network

In order to estimate the expected welfare impact of the virtual transaction and the congested network, we need to compare the results between uncongested and congested networks. As shown in the previous section, the uncongested network runs as if it is a single-bus network where all the loads and generation units are located on the single bus.

Considering what are the generally expected welfare changes due to congestion without virtual transaction may provide a good base to understand the effects of having congestion in the network on market outcomes. The congested line can inhibit the power flow from the cheaper generation to the load and the network generally has higher price relative to the uncongested case. The higher price makes the consumer purchase power at a higher cost while make producer sell at a higher price. Virtual traders can exploit the price differences more with the congested network. Overall, society loses welfare when the network is congested relative to the uncongested case.

Table 5.11 Expected welfare impact due to the introduction of optimal virtual bidding in the congested network: two-bus network

	Consumer Welfare	Producer Welfare	Total Social Welfare	Virtual Trader Welfare
Two-bus Network				
Network-wise	Decrease	Increase	Decrease	Increase
Source Bus	Increase	Decrease	Decrease	Increase
Sink Bus	Decrease	Increase	Decrease	Increase

Note: Comparing the differential qualitative welfare changes due to the introduction of optimal virtual bidding between uncongested and congested networks

In the two-bus model, when the network is congested, the qualitative nature of the optimal bidding strategy and its expected welfare impact on consumer and producer at the network level are the same as the uncongested network. However, the congested line in between bus 1 and bus 2 amplifies the expected welfare impact of the virtual transaction on the market participants.

The direction of power flow over the line 1-2 is from bus 1 to bus 2, because bus 1 has generation fleet with lower generation cost relative to the generation fleet in bus 2. In the congested network, the system operator cannot dispatch generation units in merit order. This generation inefficiency creates greater arbitrage opportunities for the financial traders to exploit, and brings greater price changes due to virtual bids relative to the uncongested case. This phenomenon amplifies the expected welfare impact in the network. Having the line 1-2 congested, consumers and society lose more while producers and virtuals gain more profit relative to the uncongested network.

When the network has a congested line, a source and a sink are buses adjacent to (next to) the congested line. The source is a bus located at the start point of the predominant power flow adjacent to the congested line. The sink is a bus located at the end point of the predominant power flow adjacent to the congested line. In the two-bus network with the power flow from bus 1 to bus 2, bus1 is the source bus and bus 2 is the sink bus.

The expected welfare impacts on market participants of the virtual transaction in the congested network vary depending on where they are located in the network. The magnitudes of expected welfare changes differ between the source and sink buses while qualitative expected welfare changes are maintained. At the sink bus, virtual transactions decrease consumer welfare and increase producer welfare while virtual trader gains more relative to the source bus in the congested network.

This heterogeneous expected welfare impact between the source and sink bus is due to the inability to dispatch the generation units at the source bus to meet the demand in the sink due to the congested line. Consequently, the generation units at the source commit less MW of energy while the generation units at the sink commit more MW of energy relative to the uncongested case.

When more generation units are committed in DA, the more expensive subset of generation units in RT is available to be online when the realized demand in RT is higher than the forecasted demand in DA. This situation in the sink bus is more favorable for the financial trader to arbitrage the price differences that bring further expected welfare changes to market participants. Hence, MW of virtual bids at sink bus is greater than source bus.

In order to estimate the expected welfare impact of virtual transactions in a congested network, we need to compare the results between uncongested and congested networks. To compare the quantitative differences, we set the uncongested as a baseline. Relative to the uncongested case, having a congested line in the network further decreases consumer and social welfare while increases producer and financial welfare. At the source bus, consumer welfare increases and producer welfare decreases because of virtual trading when the network is congested relative to the uncongested network. At the sink bus, in contrast to the source bus, expected welfare of consumers decrease and producers increase.

5.2.3 Welfare Changes due to Introduction of Optimal Virtual Bidding in a Congested Network with Loop Flow

The two-bus and three bus models have differences in generation dispatch to deal with congested network due to loop flow. In the two-bus case, when the line is congested, the buses are disconnected and the generators at each bus must meet the demand in their local bus independently. In the three-bus case, when the line is congested due to the parallel power flow from bus one to bus two, the system operator can response differently. To dispatch more volume from the generation units with lower marginal cost at bus one so they can be delivered to bus two, the system operator can dispatch more generation at bus two so the parallel power flow from bus two to bus one can lower the offset the line congestion.

The congested line can limit the power flow from source to sink in the three-bus network models, as in the two-bus model. The differences in the power flow between the two-bus and three-bus model are loop-flow. The power flow in the three-bus network are power flows not only directly via line 1-2, but also through a parallel path along line 1-2 and line 1-3 to line 3-2 due to loop flow. The relative differences of line reactance determine the magnitude of power flow on each parallel path.

Table 5.12 Expected welfare impact due to the introduction of optimal virtual bidding in the congested network: three-bus networks

	Consumer Welfare	Producer Welfare	Total Social Welfare	Virtual Trader Welfare
Three: Load Divide				
Network-wise	Decrease	Increase	Decrease	Increase
Source Bus	Increase	Decrease	Decrease	Increase
Sink Bus	Decrease	Increase	Decrease	Increase
Other Bus	Decrease	N/A	Decrease	Increase
Three: Gen Divide				
Network-wise	Decrease	Increase	Decrease	Increase
Source Bus	Increase	Decrease	Decrease	Increase
Sink Bus	Decrease	Increase	Decrease	Increase
Other Bus	N/A	Increase	Increase	Increase

Note: Comparing the differential qualitative welfare changes due to the introduction of optimal virtual bidding between uncongested and congested networks

In the electricity network, the system operator is not able to control the magnitude of power flow on the transmission lines. When the system operator plans to dispatch generation unit at bus 1, even though the capacity of line 1-2 is limited and line 1-3 is unlimited, they must consider the MW of power flow on line 1-2 and 1-3 governed by loop-flow. In this context, the congestion on the line 1-2 limits the power flow on the line 1-3. Having congestion in the three-bus network limits the merit order dispatch and may increase the cost of electricity generation and consequently, the cleared price. The qualitative expected welfare impacts of having congestion in the three-bus network are not dissimilar from the two-bus model results.

The network-wise expected welfare impacts of virtual bids on market participants from the congested three-bus load divide and the three-bus generation unit divide models are qualitatively identical to the results from the two-bus model. The different physical characteristics between two- and three-bus models do not affect the qualitative expected welfare impact.

As in the case of the two-bus model, having congestion in the three-bus networks also limit the merit order dispatch and this inefficiency creates greater arbitrage opportunities for the financial traders to exploit. The congested network also amplifies the expected welfare impact. Having the line 1-2 congested, consumer and society lose more while producer gains and virtual bids gain more profit relative to the uncongested network.

Similar to the two-bus model, the impact on market participants of the virtual transaction in the congested network varies depending on where they are located in the network in the three-bus models. While expected qualitative expected welfare changes are maintained, the magnitudes of expected welfare change depend on whether they are located at the source or sink buses.

The virtual trader uses the heterogeneous bidding behavior at the sink and the source buses. More DECs are bid at the sink bus relative to the source bus to exploit the more profitable opportunities at the sink bus where greater MW of generation units are committed in the DA market.

Relative to the uncongested case, by the financial trader, having a congested line in the network decreases consumer and total social welfare while increasing producer and financial welfare. At the source bus, due to introducing virtual, consumer welfare increases and producer welfare decreases when the network is congested relative to the uncongested case. At the sink bus, relative to the sink bus, consumer welfare decreases, and producer and financial trader welfare increases when there is a congestion due to the virtual transaction.

The three-bus models, obviously, have one more bus than the two-bus model and it is not the source or sink bus. The other bus (bus 3) can have load in the load divide model or generation unit in the generation unit divide model. The expected welfare impact of the virtual transaction at the other bus is defined by who are the market participants on that bus. If the load is located on the other bus, the expected welfare of market participants at the other bus will be decreased by virtual bids.

5.2.4 Welfare Changes due to Introduction of Optimal Virtual Bidding in an Occasionally Congested Network

A network can have a different status such as always uncongested and occasionally congested depending on the relative differences of magnitude between the MW of line capacity and the MW of power flow on that line. When the line capacity is far greater than the power flow in DA, the network will be certainly uncongested in DA and RT markets. When the line capacity is far less than the power flow in DA, the network will be certainly congested in DA and RT markets. However, when the line capacity is a little bit less than the DA power flow in the absence of the line capacity limit, the DA market will experience congestion while the RT market will have congestion occasionally, depending on the realized demand in RT. On the other hand, when the line capacity is a little bit greater than the DA power flow, the DA market will experience no congestion while the RT market may have congestion occasionally, similarly, depending on the realized demand in the RT market.

In the case of the occasionally congested transmission line, its qualitative expected welfare impact on market participants is similar to the always congested network. Hence, describing estimated qualitative and quantitative changes in bidding strategy and welfare impact in the occasionally congested case would be redundant. Instead, we focus on the trend of the quantitative change as the line capacity increases from more to less tightly constrained. The trend can be divided into two regions, where line capacity is below DA power flow and where line capacity is above DA power flow.

Table 5.13 Expected trend of welfare impact due to the introduction of optimal virtual bidding in the occasionally congested two-bus network (Congested in DA and congested part of the time in RT)

	Consumer Welfare	Producer Welfare	Total Social Welfare	Virtual Trader Welfare	Bid Quantity (MW)
Two-bus Network					
Network-wise	Decreasing	Decreasing	Decreasing	Increasing	Less DECs
Source	Increasing	Decreasing	Decreasing	Decreasing	Less DECs
Sink	Decreasing	Increasing	Decreasing	Increasing	More DECs

Note: These are the trends of expected welfare changes as the line capacity is increasing while the network is congested in the day ahead and only congested part of the time in real time.

When the line capacity is below the DA power flow and increasing from the point of the line capacity that creates congestion all the time in DA and RT towards the point of the line capacity that is MW of DA power flow, the network is always congested in DA and the chance of having congestion in RT is decreasing relative to when the line capacity is well below the MW of DA power flow, when the network is always congested in RT.

If the trend of the welfare impact is ‘Decreasing’, it represents that the welfare impact due to the introduction of optimal virtual bidding is decreasing on the specific agent along with the increasing line capacity. The start point of the trend is when the line capacity is well below the MW of DA power flow so the network is always congested in DA and RT and the end point of the trend is when the line capacity first achieves a zero shadow price, at which point the network is occasionally congested in RT.

In this case, the virtual trader's profit is increasing with the line capacity. When the MW of line capacity is getting closer to the MW of DA power flow, financial trader tends to bid more DECs at the sink and the MW of power flow in DA becomes greater than without virtual bids.

The financial trader gradually bids fewer DECs at source bus and more DECs at sink bus, and the overall volume of bids decreases. Decreasing volume of DECs at source bus shifts the magnitude of expected welfare change of market participants. Consumer welfare is negative, and the magnitude of expected welfare change increases as line capacity increases relative to the uncongested case. The producer welfare change shows the opposite trend to the consumer welfare. Relative to the uncongested case, the producer expected welfare impact of virtual trading is positive, and decreases with the increasing line capacity. Social welfare changes due to virtual trading are negative and decrease as line capacity increases.

Table 5.14 Expected trend of welfare impact due to the introduction of optimal virtual bidding in the occasionally congested two-bus network (Uncongested in DA and congested part of the time in RT)

	Consumer Welfare	Producer Welfare	Total Social Welfare	Virtual Trader Welfare	Bid Quantity (MW)
Two					
Network-wise	Increasing	Decreasing	Increasing	Decreasing	Less DECs
Source	Decreasing	Increasing	Increasing	Decreasing	More INCs
Sink	Increasing	Decreasing	Increasing	Increasing	More DECs

Note: These are the trends of expected welfare changes as the line capacity is increasing while the network is uncongested in the day ahead and only congested part of the time in real time.

When the line capacity is increasing and above the DA power flow, the network is uncongested in DA and the chance of having congestion in RT is further decreasing. In this case, the optimal bidding strategy of virtual trader differs and profits are higher when the line capacity is close to the DA power flow.

If the trend of the welfare impact is ‘Increasing,’ it represents that the welfare impact due to the introduction of optimal virtual bidding is increasing on the specific agent along the increasing line capacity. The starting point of the trend is when the line capacity is active but with zero shadow price in DA, which results in an occasionally congested line in RT, and the end point of the trend is when the line capacity is sufficient so that the line is always uncongested in DA and RT.

In the case that the MW of line capacity is getting greater than the MW of DA power flow, the financial trader tends to bid more virtual products on the network. When the line capacity is increasing and still close to the DA power flow, the profit-maximizing strategy of financial trader becomes bidding virtual supply at the source and virtual demand at the sink in order to increase the MW of power flow in DA. When the line capacity becomes far greater, the bidding volume of INCs and DECs at the source and sink bus, respectively, drops.

As the chance of having congestion in the network is decreasing as line capacity increases, the consumer, and social welfare are increasing while the producer welfare is decreasing at the aggregate network level. The financial trader tends to bid more INCs at the source bus and more DECs at the sink bus to incur congestion in DA market. At the source bus, along with the increasing line capacity, the price difference between DA and RT markets is enlarged due to the increased MW of committed and dispatched generation. Consumer welfare drops while producer and social welfare increase due to the widened price spread. At the sink bus, along with the increasing line capacity, the price differences between DA and RT become smaller because more power is flowing from the source bus. Hence, consumer and total social welfare at the sink bus are increasing while producer welfare is decreasing.

In this regard, the change in line capacity, especially when it is near to the MW of power flow without virtual bids, influences the bidding strategy and consequently the expected welfare impact on market participants. When the line capacity is near the power flow in DA, the financial trader's bidding strategy causes congestion in DA, which would not be congested without financial traders. This bidding strategy creates a distortion in the welfare of market participants that decrease the consumer and social welfare while increasing the producer and financial trader welfare.

Table 5.15 Expected trend of welfare impact due to the introduction of optimal virtual bidding in the occasionally congested three-bus networks (Congested in DA and congested part of the time in RT)

	Consumer Welfare	Producer Welfare	Total Social Welfare	Virtual Trader Welfare	Bid Quantity (MW)
Three: Load Divide					
Network-wise	Decreasing	Decreasing	Decreasing	Increasing	Less bids
Source	Increasing	Decreasing	Decreasing	Decreasing	Less DECs
Sink	Decreasing	Increasing	Decreasing	Increasing	More DECs
Other	Decreasing	N/A	Decreasing	Decreasing	Less DECs
Three: Gen Divide					
Network-wise	Decreasing	Decreasing	Decreasing	Increasing	Less bids
Source	Increasing	Decreasing	Decreasing	Decreasing	Less DECs
Sink	Decreasing	Increasing	Decreasing	Increasing	More DECs
Other	N/A	Decreasing	Decreasing	Increasing	Less DECs

Note: These are the trends of expected welfare changes as the line capacity is increasing while the network is congested in the day ahead and only congested part of the time in real time.

In the case of the occasionally congested transmission line in the three-bus models, the bidding strategy and its expected welfare impact on market participants are generally similar to the congested network case. The virtual transactions increase producer welfare and decrease consumer and social welfare while financial trader gains profit.

We can focus on the trend of a quantitative change as the line capacity increases. The trend can be divided into two segments, when line capacity is below DA power flow and when line capacity is above DA power flow.

As the line capacity gets closer to the DA power flow, financial trader tends to bid more DECs on the sink and the MW of power flow in DA becomes greater than without virtual bids. From this bidding strategy, the virtual trader is gaining more profit over the increasing MW of line capacity than when the network is always congested in RT with the line capacity well below MW of power flow in DA.

Table 5.16 Expected trend of welfare impact due to the introduction of optimal virtual bidding in the occasionally congested three-bus network (Uncongested in DA and congested part of the time in RT)

	Consumer Welfare	Producer Welfare	Total Social Welfare	Virtual Trader Welfare	Bid Quantity (MW)
Three: Load Divide					
Network-wise	Increasing	Decreasing	Increasing	Decreasing	More bids
Source	Decreasing	Increasing	Increasing	Decreasing	More INCs
Sink	Increasing	Decreasing	Increasing	Decreasing	More DECs
Other	Increasing	N/A	Increasing	Decreasing	Less DECs
Three: Gen Divide					
Network-wise	Increasing	Decreasing	Increasing	Decreasing	More bids
Source	Decreasing	Increasing	Increasing	Decreasing	More INCs
Sink	Increasing	Decreasing	Increasing	Decreasing	More DECs
Other	N/A	Decreasing	Increasing	Increasing	More INCs

Note: These are the trends of expected welfare changes as the line capacity is increasing while the network is uncongested in the day ahead and only congested part of the time in real time.

In this case, the results from the three-bus models are similar to the ones from the two-bus model. At the network level, the expected consumer and social welfare change is generally negative and decreasing while the expected producer welfare change is positive however decreasing by the virtual transaction. The virtual trader has increasing profit. As from the two-bus model, at the sink bus, the consumer tends to lose more and the producer tends to gain more relative to the market participants at the source bus.

Contrary to the above case, when the line capacity is above the DA power flow and increasing, the network is uncongested in DA and the chance of having congestion in RT is further decreasing. The virtual trader has increased profit while the line capacity is near the MW of power flow in DA.

In this case, as shown from the two-bus model results, trends of expected welfare changes differ. The qualitative changes are as before, however, consumer and society are gaining while producer and financial trader are losing expected welfare in the network over the increasing MW of line capacity. The heterogeneous impacts are generally opposite to the previous type of occasional congestion, line capacity is increasing and below DA power flow. At the sink bus, the consumer tends to gain more and the producer tends to lose more relative to the source bus.

5.3 Two- and Three-bus Models Simulation

We derive the results of expected welfare impacts from specific cases of the two- and three-bus models and the results are consistent across the cases. Nonetheless, it is yet too early to generalize the expected welfare impact by the virtual transaction in the congested network since some other network specifications may show different results.

For this reason, we will run multiple simulations with different configurations of the two- and three-bus network and compare the expected welfare impact due to the introducing optimal virtual bids and without virtual in all the randomly generated cases. Then we will derive robust expected welfare impacts on the simplified electricity network with congestion based on the wide range of randomized parameters in the simulation.

5.3.1 Two- and Three-bus Models Simulation Configuration

In order to generate random cases to run the simulation, we use the two-bus network for the base case to generate random parameters to create a diverse set of network configurations. First, the random variables from the uniform distribution of [0.3, 3] will multiply the loads in the bus 1 and bus 2, respectively (Load (‘1’) = 200 * Uniform[0.3 , 3], Load (‘2’) = 200 * Uniform[0.3 , 3]). The MW of load at bus 1 can be greater or lesser than the MW of the load in bus 2. After generating random load, we run the case without virtual trader and without line capacity limits, which is an uncongested case and measure the power flow on the line from bus 1 to bus 2.

Based on the generated specification of the two-bus network, three versions of three-bus models are generated: load divide, generation unit divide and load and generation unit divide. The load divide version is generated by separating the load at bus 1. The load in bus 1 becomes the loads in bus 1 multiplied by the random scaler from the uniform distribution of [0.1, 1] (Load(‘1’)_LoadDivide = Load (‘1’) * uniform[0.1,1]). The load in bus 3 is generated by subtracting the random scalar from 1 and multiply it to the load in bus 1 (Load(‘3’)_LoadDivide = Load (‘1’) * (1-uniform[0.1,1])).

The generation divide version is based on the generated specification of the two-bus network. The generation units in bus 1 are separated into bus 1 and bus 3. The supply function slope in DA in bus 1 in the generation unit divide model becomes the supply function slope in DA in bus 1 in the two-bus model multiply the random scaler from the uniform distribution of

[0.1, 1] (b1('1')_GenDivide = b1 ('1') * uniform[0.1,1]). The supply function slope in DA at bus 1 in the generation unit divide model is determined by subtracting the random scalar from 1 and multiplying the supply function slope in DA in bus 1 in the two-bus model (b1('3')_GenDivide = b1 ('1') * (1-uniform[0.1,1])).

The load and generation divide three-bus version is generated by combining the load divide and generation divide version. Hence, the one case from the two-bus model and three cases from the three-bus model, become a set of network cases that are generated to be compatible with each other. The market outcomes, such as price, the overall MW of virtual bids in the network, and the expected welfare impacts are identical among the four compatible cases.

The magnitude of line reactance in the generated three-bus network cases is adjusted so that they can have the same MW of power flow from bus 1 to bus 2 in the DA market without virtual trader. The congested case is defined by setting the line capacity at 80% of power flow on line from bus 1 to bus 2 in the DA market without virtual trader and congested in the RT market. We measure the expected welfare impact in both the uncongested and the congested networks.

5.3.2 Two- and Three-bus Model Simulations Results: Welfare Impacts of Virtual and Congested Network

The purpose of using a simulation approach is to check whether the results obtained from the specific configurations of the two- and three-bus network models can be extended and generalized by the randomly generated multiple cases. The previous models with a specific network with zero expected demand deviations resulted that virtual transactions decrease the expected welfare of the consumer and society, and increase producer welfare, while the virtual trader always profits. Comparing the uncongested and congested network cases indicates that the expected welfare impact of virtual transactions on market participants tend to be amplified by line congestion.

Table 5.17 Expected welfare impacts of virtual and congested networks in the Two- and Three Bus simulation

Frequency of expected welfare increase due to virtual transactions				Magnitude of expected welfare change due to virtual transactions			
Consumer Welfare	Producer Welfare	Total Social Welfare	Virtual Trader Welfare	Consumer Welfare	Producer Welfare	Total Social Welfare	Virtual Trader Welfare
CV-C: Expected welfare impact due to virtual in a congested network							
0.0%	100.0%	0.0%	100.0%	-2151	465	-1687	14
UV-U: Expected welfare impact due to virtual in an uncongested network							
0.0%	100.0%	0.0%	100.0%	-1147	400	-747	5

Note: Iteration = 100

Table 5.17 summarizes the results of simulation at the network level. According to the columns for the frequency of expected welfare increased due to virtual, the expected welfare impacts in both uncongested and congested cases are consistent with the results from the previous results of specified models. There is no case when virtual transactions in the network increase consumer and total social welfare, and interestingly virtual transactions always increase producer welfare. The virtual trader always makes profits.

The averaged magnitude of the expected welfare changes due to the virtual transaction are displayed in the right section of Table 5.17 for four versions of networks: two-bus, three-bus load divide, three-bus generation divide and three-bus generation and load divide. While the qualitative impacts are consistent across the uncongested and congested cases, the magnitudes differ. The congested line tends to magnify the expected welfare impacts that the consumer and society lose more while the producer and virtual trader gain more.

Using a simulation approach enables us to obtain robust evidence of expected welfare impacts on the different locations in the network. Among the number of buses in the network, source and sink are buses adjacent to (next to) each congested line. The source bus located at the origin of the predominant power flow adjacent to the congested line. The sink bus is a bus located at the destination of the predominant power flow adjacent to the congested line.

The results obtained from specific configurations from the two- and three-bus network models suggest that having virtual transactions in the congested network have heterogeneous expected welfare impact on the market participants adjacent to the congested line. A financial trader in the network decreases consumer welfare more at the sink bus relative to the source bus while increasing producer welfare more in the sink bus relative to the source bus.

Table 5.18 Heterogeneous expected welfare impacts of virtual in the congested network in the Two- and Three Bus simulation

	Frequency of expected welfare increase due to virtual transactions				Magnitude of expected welfare change due to virtual transactions			
	Consumer	Producer	Social	Virtual	Consumer	Producer	Social	Virtual
CV-C: Expected welfare impact due to virtual in a congested network								
Network	0.0%	100.0%	0.0%	100.0%	-2151	465	-1687	14
Source	2.3%	97.5%	11.8%	99.3%	-340	111	-230	2
Sink	0.0%	100.0%	3.0%	100.0%	-1622	310	-1312	10
Other	0.0%	99.5%	60.0%	100.0%	-378	88	-194	3

Note: Iteration = 100, Source and sink are buses adjacent to (next to) each congested line. The source is a bus located at the start point of the predominant power flow adjacent to the congested line. The sink is a bus located at the end point of the predominant power flow adjacent to the congested line.

The simulation results also show that the expected welfare impact of the virtual transaction is heterogeneous in the congested network, supporting the results from the specified two- and three-bus models. The expected impacts for the consumer and the producer are clear. Virtual transaction generally decreases the consumer welfare while increasing the expected producer welfare regardless of their locations in the network. At a sink bus, the producer and financial trader tend to gain more while consumer and society tend to lose more relative to the source bus.

For the other bus, in the three-bus networks, we randomly distribute the magnitude of load, the slope of generation unit, or both to be located at that bus. Virtual transactions tend to decrease expected consumer welfare, and increase the expected producer welfare. The social welfare at that bus, the magnitude of the expected social welfare impact is negative and more than half of the time it is decreased. The financial trader makes a profit on the other bus.

5.3.3 Two- and Three-bus Model Simulations Results: Welfare Impacts of Virtual and Loop Flow

One of the benefits of the simulation approach is that it permits quantification of the expected welfare impact. It allows comparison of the magnitude of welfare impact between different types of models. In the deterministic case, the qualitative welfare impact of introducing virtual trading in the two- and three-bus networks are identical to the market participants.

In the overall network, the welfare impact of introducing optimal virtual bidding on market participants is not vastly different due to having loop flow in the network. The slightly amplified welfare impacts on market participants may be explained by having an additional bus to locate physical assets. In the three-bus network with generation divide, the producer has reduced positive welfare since there are more generation assets that can be utilized in the congested network. In the three-bus network with load divide, the consumer loses more due to one more load node with stochastic demand in the real-time market.

Table 5.19 Expected welfare impacts of virtual in the congested network in the Two-Bus (No loop flow) and Three Bus (Loop flow) simulation

	The magnitude of expected welfare change due to virtual transaction at the network level			
	Expected Consumer Welfare	Expected Producer Welfare	Expected Total Social Welfare	Expected Virtual Trader Welfare
CV-C: Expected welfare impact due to virtual in a congested network				
Two	-2103	439	-1664	15
Three: Gen divide	-2123	444	-1679	15
Three: Load divide	-2256	503	-1753	13
Three: Gen + Load divide	-2123	473	-1651	14

Note: Iteration = 100

5.4 Two- and Three-bus Models Result Summary

In the cases of the uncongested network, the expected welfare impacts of the optimal virtual bids on market participants show consistent results with the results of algebraic analysis of the single bus case. In all cases, the optimal virtual bids obtain surplus from its transactions. With the zero expected demand deviation, the consumer loses and the producer gains.

In both two-bus and three-bus models the expected welfare impact is qualitatively identical to the uncongested case. The impact of virtual bids in congested two-bus and three-bus cases maintain the directional change in the expected welfare of market participants, yet amplified, relative to the uncongested cases. In the case of congestion, the system operator has a different strategy of dispatching generation in merit order to deal with line congestion in the two-bus and three-bus cases.

The congestion happens all the time when the line capacity is well below the expected range of uncertain demand, while the congestion happens at no time when the line capacity is well above the expected range of uncertain demand. Congestion happens occasionally when the line capacity lies in the middle of the expected range of uncertain demand. In this case, the qualitative welfare impact is maintained while the magnitude of the impact is changing over the incremental line capacity change.

Table 5.20 Expected trend of welfare impact due to the introduction of optimal virtual bidding in two-bus network

	Consumer Welfare	Producer Welfare	Total Social Welfare	Virtual Trader Welfare
Network-wise				
Congested	More Decrease	More Increase	More Decrease	More Increase
Occasionally Congested: Capacity < Power Flow in DA	Decreasing	Decreasing	Decreasing	Decreasing
Occasionally Congested: Capacity > Power Flow in DA	Increasing	Decreasing	Increasing	Decreasing
Uncongested	Less Decrease	Less Increase	Less Decrease	Less Increase
Source Bus				
Congested	More Decrease	More Increase	More Decrease	More Increase
Occasionally Congested: Capacity < Power Flow in DA	Increasing	Decreasing	Decreasing	Decreasing
Occasionally Congested: Capacity > Power Flow in DA	Decreasing	Increasing	Increasing	Decreasing
Uncongested	Less Decrease	Less Increase	Less Decrease	Less Increase
Sink Bus				
Congested	More Decrease	More Increase	More Decrease	More Increase
Occasionally Congested: Capacity < Power Flow in DA	Decreasing	Increasing	Decreasing	Increasing
Occasionally Congested: Capacity > Power Flow in DA	Increasing	Decreasing	Increasing	Increasing
Uncongested	Less Decrease	Less Increase	Less Decrease	Less Increase

In the occasionally congested case when the line capacity is lower than power flow in the day ahead market, the congestion happens more often than when the line capacity is greater than the power flow in the day ahead market. If the line capacity is far greater than the power flow in the day ahead market and large enough to not to constrain the power flow, the network is uncongested all the time. The expected probability of congestion by demand uncertainty and line capacity change the optimal bidding strategy for the financial traders, and it shows a certain trend of welfare change in magnitude while the qualitative welfare impact is maintained.

Table 5.21 Welfare impacts of virtual in the congested network in the Two- and Three Bus Simulation

	Frequency of expected welfare increase due to virtual transactions				Magnitude of expected welfare change due to virtual transactions			
	Consumer	Producer	Social	Virtual	Consumer	Producer	Social	Virtual
CV-C: Expected welfare impact due to virtual in a congested network								
Network	0.0%	100.0%	0.0%	100.0%	-2,151	465	-1,687	14
Source	2.3%	97.5%	11.8%	99.3%	-340	111	-230	2
Sink	0.0%	100.0%	3.0%	100.0%	-1,622	310	-1,312	10
Other	0.0%	99.5%	60.0%	100.0%	-378	88	-194	3
UV-U: Expected welfare impact due to virtual in an uncongested network								
Network	0.0%	100.0%	0.0%	100.0%	-1,147	400	-747	5

Note: Iteration = 100, Source and sink are buses adjacent to (next to) each congested line. The source is a bus located at the start point of the predominant power flow adjacent to the congested line. The sink is a bus located at the start point of the predominant power flow adjacent to the congested line.

We derive the estimation from specific cases of the two- and three-bus models and the results of expected welfare impacts are consistent across the cases. However, they may not support to obtain robust expected welfare impact by the virtual transaction in the congested network since some other network specifications may show different results.

For this reason, we implement multiple simulations with different configurations of the two- and three-bus network and compare the expected welfare impact between the uncongested and uncongested cases in all the randomly generated cases. Then we derive robust expected welfare impacts on the simplified electricity network with congestion, which are consistent with the results from the specific cases.

Overall, at the network level, having virtual transactions generally decreases expected consumer and social welfare while increasing expected producer welfare. Having congestion in the network amplifies the expected welfare impact of the virtual transaction, the consumer and society lose more and the producer gain more. When the network is congested, the expected welfare impact is heterogeneous and depending on where the market participants are located relative to the congested line. At the sink bus, the consumer and society tend to lose more while the producer tends to gain more relative to the source bus. At the other bus, in general, the consumer loses and producer generally loses while the expected welfare impact on the society is ambiguous.

CHAPTER 6. WELFARE ANALYSIS IN THE ISO-NE TEST NETWORK

The previous chapter analyzed two- and three-bus networks using simulation to estimate the welfare impacts of the introduction of virtual transactions in a congested network. The two-bus network is the simplest network that includes transmission. The three-bus network is the simplest network that can incorporate a loop, and hence where Kirchhoff's Laws governing loop flows in an electrical network apply.

Using these simple electricity networks provides a "sandbox" to facilitate understanding the impact of the introduction of virtual transactions in an electricity network with the capacity-constrained transmission. Based on the simulation analysis, we estimated the welfare impacts of the virtual transaction on the market participants both in aggregate and depending on their location in the network relative to the congestion.

However, real electricity networks are far more complicated than our sandbox models, with more buses and transmission lines and many possible patterns of congestion. The financial trader needs to design an optimal bidding strategy for virtual products that account for the complexities of the network and the potential for congestion from a holistic perspective.

The principal question at hand is whether the lessons regarding virtual trader behavior and the impacts on market participants' welfare extend from the simple networks to larger, more complex networks. To facilitate addressing this question, we employ a stylized version of the network of the ISO-NE in a simulation context under alternative loadings.

6.1 ISO-NE Test System

The organization of this section is as follows. The first subsection addresses the network test system configuration in terms of zonal, aggregate transmission lines; zonal aggregate supply functions for both the cases when all generators are available (i.e. day ahead) and when only peaking units are available (i.e. real-time); and independently observed of zonal demand.

6.1.1 ISO-NE Test System Configuration

We will use the ISO-NE Test System developed by Krishnamurthy, Li, and Tesfatsion (2016) as the basis for our analysis. The test system models the ISO New England (ISO-NE) wholesale electricity market spanning a mix of generating companies and load-serving entities that operate in an eight-zone aggregated transmission grid. The generation, load, and transmission line attributes in the test system are taken from ISO-NE data developed by Krishnamurthy, Li, and Tesfatsion (2016).

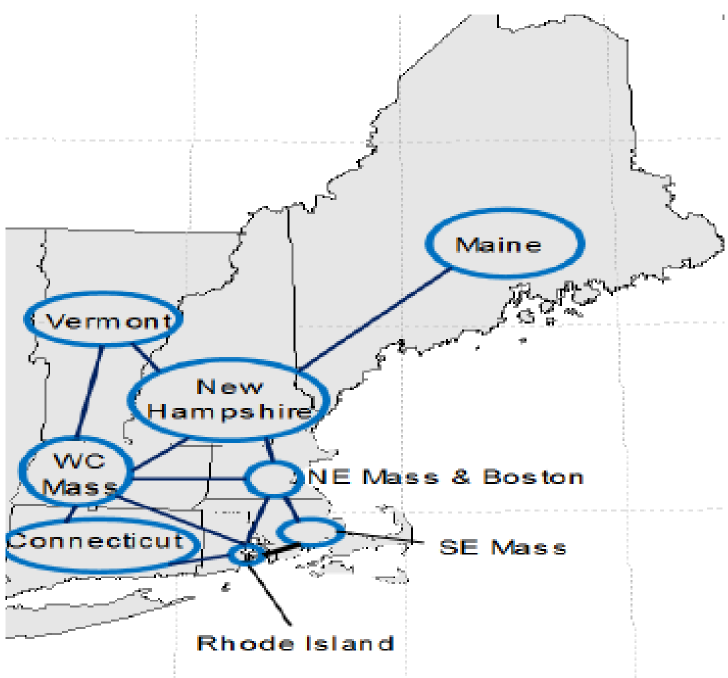

Figure 6.1 The eight-zone ISO-NE Test System (Krishnamurthy, Li, and Tesfatsion, 2016)

The ISO-NE Test System suits the purposes of this study, for several reasons. First, it is substantially larger than our sandbox models. Second, it is relatively complex, incorporating a number of loops, making loop flow a potentially important phenomenon. Third, while highly aggregated it has a clear basis in the real world, and fourth, unlike many other ISO/RTO-scale systems, the basic model is open source and readily accessible for replication purposes.

ISO-NE is part of the Northeast Power Coordinating Council (NPCC) reliability region. The states covered by ISO-NE are aggregated into eight load zones: Connecticut (CT), Maine (ME), New Hampshire (NH), Rhode Island (RI), Vermont (VT), Northeastern Massachusetts/Boston (NEMA/BOST), Southeastern Massachusetts (SEMA) and Western/Central Massachusetts (WCMA). Reflecting this configuration, the eight zones in the ISO-NE Test System are connected by twelve transmission lines as indicated in Figure 6.1.

Table 6.1 Transmission line benchmark values for the ISO-NE Test System (Krishnamurthy, Li, and Tesfatsion, 2016)

From Zone	To Zone	Resistance (ohms)	Reactance (per unit)
ME	NH	19.09	0.05
VT	NH	16.6	0.04
VT	WCMA	24.9	0.06
WCMA	NH	14.28	0.03
NEMA/BOST	WCMA	13.28	0.03
NEMA/BOST	NH	10.46	0.02
NEMA/BOST	SEMA	4.98	0.01
WCMA	CT	4.98	0.01
WCMA	RI	10.79	0.03
NEMA/BOST	RI	6.64	0.02
CT	RI	10.62	0.03
SEMA	RI	3.32	0.01

The structural attributes, resistance, and reactance benchmark values for the 12-line test system network are developed by Krishnamurthy, Li, and Tesfatsion (2016) and are set to approximate the function of the aggregation of the actual lines between zones. The key factors that determine these values include the length of each line, conductor type, conductor bundling and transposition (Krishnamurthy, Li, and Tesfatsion 2016). The calculated benchmark values for the transmission lines are represented in Table 6.1.

The load scenarios for our simulation analysis are based on ISO-NE March hourly load data for 2004-2006 (Krishnamurthy, Li, and Tesfatsion, 2016). These data are reported separately for each of ISO-NE's eight load zones. Summary statistics for these 90 hourly load scenarios are displayed in Table 6.2.

Table 6.2 Load data summary (Krishnamurthy, Li, and Tesfatsion, 2016)

Name	Avg	S.D	Min	Max
CT	1,343	87	1,172	1,521
WCMA	1,337	102	1,090	1,609
VT	683	42	598	786
SEMA	3,715	294	2,870	4,493
RI	923	74	707	1,107
NEMA/BOST	1,730	137	1,353	2,173
NH	2,068	174	1,576	2,535
ME	2,855	221	2,291	3,541

ISO-NE has a total installed capacity of 32,000MW with 151 thermal generation units providing roughly 88% of the total capacity. In the study of Krishnamurthy, Li, and Tesfatsion, (2016), all non-thermal generation units are excluded. In addition, 76 of the 151 thermal generation units were selected in their study for inclusion in the benchmark generation mix with each treated as an independent generator in there study. These 76 generators have a combined installed generation capacity of 23,100MW and account for 72% of the actual ISO-NE capacity. In Krishnamurthy, Li, and Tesfatsion, (2016), the dispatch cost functions for each of the 76 generators are assumed to be quadratic and the parameters were derived from ISO-NE generation block-offer schedule data by fuel type.

Table 6.3 Generator capacity by type and zone

	Capacity (MW)		
Zone	All	Baseload	Peaker
CT	5,694.6	2,860.4	2,834.2
WCMA	1,227.7	144.4	1,083.3
VT	620.2	620.2	0.0
SEMA	2,962.7	684.7	2,278.0
RI	5,026.1	1,099.5	3,926.6
NEMA/BOST	0.0	0.0	0.0
NH	2,247.6	1,339.4	908.2
ME	5,321.4	311.8	5,009.6

Note: VT does not have peaker generators and NEMA/BOST does not have generators in the Test System.

We divide the benchmark generators into two groups, peakers and base (non-peakers). The peakers are generators fueled by natural gas or oil. The base generators are fueled by nuclear or coal. Using quadratic cost functions of each generator in each zone, we developed the zonal stacked value of zonal generation cost to meet the fixed demand in that zone by solving cost minimization (economic dispatch) problem for each zone. Then, we fit either the exponential or the quadratic functional form for aggregated peakers and aggregated base (non-peakers), at the zonal level. We fit a zonal curve to the aggregated cost function for all the generators located in that specific zone.

Supply curves for all generation units at the zonal level are fitted to this data to reflect the supply response in the day ahead market. Additional supply curves only for peaking units at the zonal level are fitted to the regional price/quantity data to reflect the supply response in the real-time market. Tables 6.3 and 6.4 represent the capacity and the fitted supply function for each type of generator at the zonal level.

Table 6.4 Generator supply functions and estimated parameters by type and zone

	All			Peaker		
	Type	parameter_a	parameter_b	Type	parameter_a	parameter_b
CT	Exp	5,715	0.0008	Quad	42	0.0454
WCMASS	Exp	13,061	0.0025	Exp	22,917	0.0024
VT	Quad	11	0.0002	N/A	N/A	N/A
SEMASS	Exp	13,174	0.0011	Quad	14	0.0474
RI	Exp	5,274	0.0009	Exp	7,749	0.0010
NEMA/ BOST	N/A	N/A	N/A	N/A	N/A	N/A
NH	Exp	5,037	0.0010	Quad	14	0.0265
ME	Exp	18,294	0.0007	Exp	22,301	0.0007

Note: Exp is Exponential and Quad is Quadratic. VT does not have peaker generators and NEMASSBOST does not have generators in the Test System. When the type is quadratic, the fitted function with generation g (MW) is parameter_a + parameter_b*g^2. When the type is exponential, the fitted function is parameter_a*(exp(parameter_b*g)-1)

6.1.2 ISO-NE Test System Modeling

In order to assess the results of the welfare impacts of introducing optimal virtual bidding obtained from the simpler electricity network models, we adapt the electricity network test case based on the structural attributes and supply/demand data of ISO New England.

First, we need to incorporate DA and RT market models using the different functional forms of supply function and different type of generators, particularly pasting Real-Time inverse supply function to the generated DA supply function. For each zone, the cleared MW of generation in the DA market set the zonal DA inverse supply function. Fitting the parameters for the DA inverse supply function uses all the generating units. Fitting the parameters for the peaker-only inverse supply function in RT uses peaker generation units.

The inverse supply function for the Real-Time market is identical to the inverse supply function in DA when the realized RT demand is less than forecasted DA demand. The RT inverse supply function is vertically shifted so that at the cleared DA MW quantity the function is continuous. To the left of the cleared DA quantity, the functional form is identical to the DA inverse supply, while to the right of the DA cleared quantity, the functional form is identical to the RT inverse supply that has been shifted vertically to get continuity. Note that this function, while continuous, is not differentiable at the DA cleared quantity. The continuous inverse supply function in RT is vertically shifted peaker-only inverse supply function on top of (right next to) the DA inverse supply function when the realized RT demand is greater than forecasted DA demand.

If RT demand lies below the DA cleared quantity, then the DA supply function defines the change in price associated with the demand deviation. This is because the cleared DA units can simply be dispatched downward in merit order. However, if RT demand is above DA cleared demand, then additional units must be dispatched, and in RT only peaking units are available for dispatch in addition to those dispatched in the DA market. Thus, if RT demand lies above DA cleared quantity, then the pasted RT supply function defines the change in price associated with the demand deviation.

For the case of demand deviation, we focus on the case of a zero expected demand deviation case. We use empirical observation of demand from hour 19 in March over years from

2004 to 2006 for the real-time demand with the mean of the observations defining the DA demand. Each of these 90 observations was assigned an equal probability of 1/90 in the real-time market.

The revised bilevel formulation and the KKT reformulation, based on the formulations for two- and three-bus models, to incorporate the different functional forms of supply function and different type of generators is presented in Appendix C.

6.1.3 ISO-NE Test System Example Case: Uncongested Network

The initial scenario for the test system is based on the situation where there is no congestion. Setting the capacity of each line so large that the capacity constraints cannot be binding results in an uncongested network, which functions the same as a single-bus network where all the generation units and loads are located at the same place.[3] When the expected demand deviation between DA and RT is zero, the optimal bidding strategy for the financial trader is to bid virtual demands, or DECs.

Table 6.5 displays the results of the uncongested ISO-NE Test System. At the network level, having a virtual transaction on the uncongested network decreases consumer welfare and total social welfare while increasing producer welfare. These results also apply at the bus level, although total social welfare at the bus level is heterogeneous, depending on the magnitude of the zonal demand and generation supply function at that bus.

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[3] In order to make the network uncongested, we impose very large line capacity and the network has no chance to be congested. We could lower the load in DA and RT, however that makes it hard to make a comparison of the welfare impact between congested cases.

Table 6.5 Quantitative welfare impact of introducing virtual bidding in the uncongested network

Zone	Expected Consumer Welfare	Expected Producer Welfare	Expected Total Social Welfare	Expected Virtual Trader Welfare	Expected Bid Quantity (MW)
Network Total	-2,667	2,580	-87	26	-10
CT	-245	556	312	3	-1.2
WCMA	-243	146	-98	3	-1.2
VT	-124	113	-11	3	-1.2
SEMA	-676	331	-345	3	-1.2
RI	-168	573	405	3	-1.2
NEMA/BOST	-315	N/A	-315	3	-1.2
NH	-376	410	34	3	-1.2
ME	-520	451	-68	3	-1.2

6.1.4 ISO-NE Test System Example Case: Congested Network

In this section, we compare the results between no congestion and with congestion where the line from VT to NH is congested. Table 6.6 presents results parallel to those of Table 6.5 but in the case where the network is congested. When this line is congested, the “source bus” and “sink bus” are VT and NH, respectively. That is, the source bus for a particular congested line is defined to be at the origin of the flow on the line, and the sink bus is defined to be at the destination of the flow on the line.

At the aggregate network level, the consumer and society lose more due to the introduction of virtual trading while the producer and virtual trader tend to gain more with congestion in the network relative to the uncongested network. In addition, the total MWs of virtual bids is greater in the congested case than in the uncongested case. These results are consistent with the sandbox models.

At the bus level, first, we can compare the welfare impacts of the introduction of virtual trading on market participants and the bidding strategy between the source bus (VT) and the sink bus (NH), relative to the uncongested case. In the congested case, the consumer and society tend to lose more while the producer tends to gain more at the sink bus by introducing virtual transactions, again relative to the uncongested case. This is also consistent with the previous results from the simple network models where we found that the relative location in the congested network results in that differences in the magnitude of welfare change of the market participants.

Table 6.6 Quantitative welfare impact of having a virtual trader in the congested network relative to no virtual case: Line from VT to NH is congested

Zone	Expected Consumer Welfare	Expected Producer Welfare	Expected Total Social Welfare	Expected Virtual Trader Welfare	Expected Bid Quantity (MW)*
Network	-5,261	3,779	-1,482	82	-12
CT	250	-560	-310	1	0.6
WCMA	249	-148	101	1	0.6
VT (Source)	-562	511	-51	39	-4.2
SEMA	-1,977	954	-1,023	19	-3.0
RI	-300	1,002	703	4	-1.3
NEMA/BOST	-771	N/A	-771	4	-1.3
NH (Sink)	-754	821	66	4	-1.3
ME	-1,396	1,199	-197	11	-2.2

Positive MW: INC / Negative MW: DEC

The bidding strategy at the source and sink buses, however, differ from the sandbox models. Recall that with the sandbox models, virtual trader bids greater MW of DECs at the sink relative to the source. In the ISO-NE Test Case, the virtual trader bids a greater MW quantity of DECs at the source rather than sink. The differences in the optimal bidding strategy are a result of asymmetries in the network due to the presence of congestion. As shown in Figure 6.1, nodes are interconnected to other buses in an asymmetric way – that is, some buses are directly connected via a transmission line, while others must wheel power through intervening zones. For example, VT is connected to WCMA and NH while NH is connected to VT, WCMA, ME, and NEMA/BOST. The fact that the VT and NH zones have a common connected bus, WCMA, and the complexity of loop flow due to the secondary connections between WCMA, VT, WCMA, ME impact the optimal profit-maximizing bidding strategy.

6.2 ISO-NE Test System Simulation

In the sandbox models, the congestion can only occur on a single line in the network and having congestion in the network amplifies the welfare impacts of the virtual transactions for the market participants. The consumer and society lose more while producers and virtual traders bid more and gain more in order to exploit the network with congestion.

We derive the results of expected welfare impacts from specific cases using the ISO-NE test system with a specified single line. Nonetheless, it is yet too early to generalize the expected welfare impact by the virtual transaction in the congested network since some other network specifications may show different results. Due to the complexity of the network with multiple transmission lines connecting zones, multiple lines can be congested at the same time.

For this reason, we run multiple simulations with different configurations of demands and transmission line capacities of the ISO-NE test system and compare the expected welfare impact between the introducing optimal virtual bids and without virtual in all the randomly generated cases. Then we will derive robust expected welfare impacts on the simplified electricity network with congestion.

6.2.1 ISO-NE Test System Simulation Configuration

In order to generate random cases to run the simulation, we use the ISO New England aggregated network structure and parameters and load data developed by Krishnamurthy, Li, and Tesfatsion, (2016). We use all 12 aggregate lines with benchmark reactance parameters given in Table 6.1. By setting the line capacities so large that they cannot be constrained, the base case reflects zero congestion. Each zone has fitted inverse supply functions for all generation units and the peaker generation units using either an exponential or a quadratic form as specified in Table 6.4.

As in the two- and three-bus simulation, we randomly generate parameters to adjust loads and line capacities based on the base ISO-NE test network to create a diverse set of network configuration. We generate multiplicative scale factors independently for each zone and transmission capacity so they can be adjusted independently. First, the random scalars from a uniform distribution on [0.8, 1.2] are generated independently and multiplied by the loads in the 8 zones. The multiplicative factors are applied to the entire empirical distribution of RT loads and then the mean of the distribution is calculated to serve as the DA load. The MW of the load distribution at any bus can be greater or less than the MW of the base case.

After generating random loads, we solve the DA and RT dispatch problems without virtual trading and without line capacity limits. This gives results for an uncongested case, and provides DA and RT generation and power flows as well as the expected welfare of the consumer, producer, virtual and society taking into account the DA market and the RT market. Using this information, the congested cases are generated by randomly generating line capacity.

We generate the random multiplicative factors from the uniform distribution of [0.5, 2]. Then, multiply the generated factors independently to the base line capacities, MW of power flows in DA with the virtual transaction in the uncongested case. If one or more lines are congested in RT with virtual and the optimization problem is feasible, we measure the expected welfare impacts for consumers and generators in both the uncongested and the congested cases by comparing expected welfare between introducing optimal virtual bids and without virtual bids. If the DA or any of the RT dispatch problems is infeasible or the network is not congested in any of the RT dispatch problems with the adjusted line capacities, the case is omitted from the simulation results.

6.2.2 ISO-NE Test System Simulation: Welfare Impacts of Virtual and Congested Network

Table 6.7 Simulated expected welfare impacts of introducing virtual trading in congested and uncongested networks in the ISO-NE Test Case

Frequency of welfare increase due to introduction of virtual trading				Average magnitude of welfare change due to the introduction of virtual trading			
Consumer	Producer	Society	Virtual	Consumer	Producer	Society	Virtual
CV-C: Expected welfare impact due to virtual in a congested network							
0%	100%	9%	100%	-6,121	5,108	-1,012	123
UV-U: Expected welfare impact due to virtual in an uncongested network							
0%	100%	0%	100%	-2,730	2,637	-93	26

Note: Iteration = 1000. Feasible cases = 130, Infeasible cases = 865, Uncongested cases = 5, Congested lines = 326. (Average congested lines per feasible case: 2.5)

The results from the sandbox models suggest that virtual transactions decrease the welfare of consumers and society, while increasing producer welfare. Virtual traders always profit, and hence their welfare increases relative to non-participation in the market. Comparing the uncongested network and congested network cases indicates that the welfare impacts of virtual transactions on market participants tend to be amplified by the congestion in the network.

Table 6.7 summarizes the results of the ISO-NE Test System simulation at the network level. The qualitative welfare impacts in both the uncongested and the congested cases are consistent with the results observed from the sandbox models. There are no cases where introducing virtual transactions in the network increases consumers’ expected welfare, and in every case introducing virtual transactions increases producers’ expected welfare. In addition, the virtual trader always achieves positive expected profits. The impact on social welfare at the network level, however, is ambiguous in this more complicated network. While the average magnitude of the expected welfare impact on society due to the introduction of virtual transactions is negative, expected social welfare is increased in a modest number of congested cases amounting to less than 10% of the instances.

6.2.3 ISO-NE Test System Simulations Results: Nodal Welfare Impacts Due to Introducing Virtual Transactions in a Congested Network

The results obtained from the two- and three-bus network sandbox models suggest that the welfare impact due to introducing virtual transactions in the congested network is different at the nodal level.

Each congested line has source and sink buses defined by the dominant direction of flow, and with multiple congested lines, some buses can be both a source and a sink. That is, a bus can be a sink bus for one congested line and a source bus for another congested line.

When a bus is not directly adjacent to the congested line (call these “other” buses), introducing virtual transactions tends to decrease consumer welfare and increase producer welfare at that bus. Expected social welfare on these other buses decreases most of the time.

Introducing virtual transactions generally decreases expected consumer welfare while increasing expected producer welfare in most locations in the network. At a sink bus, due to the introduction of the optimal virtual bidding, the producer, and financial trader tend to gain more while the consumer and society tend to lose more relative to the source bus. The frequency of the cases of increased expected welfare, however, suggests that the nodal welfare impacts in the complicated network may be less clear than in the simpler sandbox networks.

Table 6.8 Nodal welfare impacts of virtual in congested network in the ISO-NE Test Case simulation

	Frequency of expected welfare increase due to virtual transactions				Magnitude of expected welfare change due to virtual transactions			
	Consumer	Producer	Social	Virtual	Consumer	Producer	Social	Virtual
CV-C: Expected welfare impact due to virtual in a congested network								
Network	0%	100%	9%	100%	-6,121	5,108	-1,012	123
Source	19%	78%	45%	98%	-523	930	400	7
Sink	6%	97%	17%	100%	-2,229	1680	-668	67
Both	12%	83%	15%	98%	-888	346	-577	18
Other	0%	100%	9%	100%	-765	730	-127	15

Note: Iterations = 1000. Feasible cases = 130, Infeasible cases = 865, Uncongested cases = 5, Congested lines = 326. (Average congested lines per case: 2.5) Source and sink are buses adjacent to (next to) each congested line. The source is a bus located at the origin of the predominant power flow adjacent to the congested line. The sink is a bus located at the destination of the predominant power flow adjacent to the congested line. “Both” indicates a bus located as sink and source both. Source buses = 190, Sink buses = 186, Both buses = 69.

While we can find differential welfare impacts of market participants between the source and sink buses, there may be a confounding effect due to the complexity of the network and simultaneous existence of multiple congested lines. Using the network structure where Maine (ME) is isolated from the other network nodes except New Hampshire (NH), as represented in

Figure 6.1, we can have a clearer view of the differential welfare impact depending on the location of the network.

When there is a congestion between ME and NH, the power flows from ME to NH and thus ME is the source bus and NH is the sink bus. To isolate potential confounding effects, we focus only on the cases when there is congestion on the ME-NH line and ME is the source and NH is the sink, thus excluding cases where NH is both a source and sink.

Table 6.9 Expected welfare impacts of introducing virtual transactions in the ISO-NE Test Case simulation when the line between ME and NH is congested and ME is source bus and NH is sink bus

	Frequency of expected welfare increase due to virtual transactions				Magnitude of expected welfare change due to virtual transactions			
	Consumer	Producer	Social	Virtual	Consumer	Producer	Social	Virtual
CV-C: Expected welfare impact due to virtual in a congested network								
Source	26%	74%	74%	92%	2	-91	-54	0.2
Sink	4%	94%	4%	100%	-681	426	-255	8.4

Note: # of cases: 23

The results from the ME and NH line congestion cases are generally consistent with the sandbox models. At the sink bus, the consumer tends to lose more often and by a greater expected magnitude, while the producer and virtual trader tend to gain more often and with a greater expected magnitude, relative to the source bus. The virtual trader, while they usually get an expected welfare gain at both source and sink, they occasionally make losing bids on the source node. These are usually INCs (virtual supply), that increase the chance of congestion, thus creating greater increases in welfare gains at the sink node.

6.3 ISO-NE Test System Simulation Summary

In this chapter, we extend the form of the network from the sandbox models to a more realistic one by adapting the ISO-NE Test System and run the simulation for the randomly generated cases. Overall, the welfare impacts of the virtual transaction on the market participants are consistent with the results of the sandbox models and the closed form solution for the single bus case. On this eight bus and twelve line network based on an aggregation of the ISO-NE network configuration and observed data, introducing virtual trading generally decreases expected welfare for the consumer and society while increasing expected welfare for the producer at the network level in both uncongested and congested networks.

Relative to the uncongested case, having congestion in the network generally amplifies the expected welfare impacts due to the introduction of virtual transactions, with consumers and society losing more expected welfare while the producers and virtual traders gaining more expected welfare with congestion. In the congested network, the nodal welfare impact is heterogeneous by location relative to the location(s) of the congested line(s) as well as the network topology. At the sink buses, the consumer and society tend to lose more while the producer tends to gain more relative to the source bus due to the introduction of the optimal virtual bidding. Expected social welfare goes down the majority of time at the network level and nodal level due to the introduction of the optimal virtual bidding.

CHAPTER 7. CONCLUSION AND DISCUSSION

In the interest of improving the performance of wholesale electricity markets, virtual financial products have been introduced. Virtual bids are purely financial instruments that are used to hedge or speculate on the differences between forward and spot prices in a two-settlement electricity market. While there is evidence that virtual bidding can induce price convergence across the day ahead and real-time markets, other research has shown that the impacts of virtual bidding on the welfare of market participants are less clear.

The net revenue from the virtual transaction for the 2012-2015 planning years was $789 million in the PJM Interconnection. With such a large amount of money at stake, it is critical to verify that virtual transactions bring benefits to the market from a societal perspective – that is that social welfare is increased. Although price convergence is a convenient and commonly used measure for electricity market efficiency, price convergence may not increase welfare for the market participants in all cases.

The problem is that, while virtual bidding may narrow the expected price gap between forward and spot markets, if it does so by inflating both prices, then electricity consumers may not be better off. Although some work has analyzed the impacts of virtual transactions on welfare for the electricity market participants, that work provides an incomplete assessment, ignoring some important aspects of the electricity market system. In particular, Giraldo et al. (2017) ignored the electricity network. This is an important omission because it ignores the impacts of congestion and the physical laws governing electricity flows.

Hence, the objective of this research is to improve our understanding of the effects of virtual transactions on electricity market performance using models that explicitly include the network as well as relationships that reflect the physical properties of electricity flows through a network (i.e. loop flow). The core research question is: what impact does network congestion have on the welfare shifts caused by the participation of financial virtual traders?

This study employs models having an explicit network to analyze the welfare changes of electricity market participants in a network constrained multi-settlement electricity market. Integrating the network in the model enables a comparison of welfare changes between the simpler network-free models and a network-based model with the possibilities of line congestion and an explicit treatment of loop flow.

Using an analytical approach in a stylized single bus case, we derived closed-form solutions for the optimal virtual bidding strategy and the expected welfare impact on the market participants due to the optimal virtual bids. The interesting market situation for the system operator is having the zero expected demand deviation that the forecasted demand in DA market is the same as the expected realized demand in RT market. In this case, the optimal strategy for the financial trader is bidding virtual demand, DECs, and it increases expected producer welfare while it decreases expected consumer welfare.

An essential component ignored in the electricity market literature concerning the impact of the virtual transaction is transmission line congestion. The electricity market is a network with bus and they are connected by the transmission line. Another important component is loop flow governs the power flow through transmission lines.

When no transmission constraints are binding, power flow moves freely, and assuming no line losses, only a single price is needed to value electricity throughout the entire network. That is commonly not the case due to the limited transmission line capacity. When the MW of power flow in the transmission line reaches the line capacity, the line cannot move additional power through that line, and we call it congested. The congested line cancels ideal merit-order dispatch and limits additional power flow between buses.

Using stylized two- and three-bus models, we estimated and compared the differences in welfare impact due to the introduction of virtual transactions between uncongested and congested networks as well as its heterogonous impact on the different buses due to their location within the network. At the network level, congested lines amplify the welfare change due to introducing virtual transactions. When the network has a congested line, source and sink are at opposite side of the congested line. The source is located at the origin and the sink is located at the destination of the predominant power flow on the congested line. At the sink bus, the consumer and society tend to lose more while the producer tends to gain more relative to the source bus.

The results of welfare impacts of two- and three-bus models are consistent across the different cases of expected demand deviation. Nonetheless, the results from a handful of deterministic cases do not represent various forms of network configuration and it makes it hard to generalize the results. For this reason, we implement simulation with different random configurations of the two- and three-bus network and compare the welfare impact between the uncongested and congested cases in all the randomly generated cases. Then we could derive robust welfare impacts on the simplified electricity network with congestion, which are consistent with the results from the specific cases.

However, real electricity networks governed by ISO and RTOs are more complicated with more buses and transmission lines relative to the simple two- and three-bus network models. In this regard, we adopt the simulation approach using ISO-NE test network and derive consistent result with the simple network models.

The sandbox and ISO-NE test case simulations results suggest that price convergence occurs with optimal virtual bidding, which is consistent with existing literature. The prices throughout the network in both forward and spot markets are affected, and there are welfare transfers among producers, consumers, and virtual traders relative to the market equilibrium without virtual bidding. Results indicate that optimal virtual bidding tends to decrease consumer welfare when the expected demand is equal to the cleared demand in the forward market, and congested lines in the network can magnify the welfare changes while virtual traders essentially always benefit.

Furthermore, the welfare impacts on market participants are not homogenous throughout the network. That is, some generators may benefit while others lose, and the same is true for load-serving entities. The impacts are heterogeneous depending on where the agents are located in the network relative to the congested line. The one constant is that virtual traders essentially always benefit because the financial trader can always opt out. These implications should be considered in the design of regulations governing virtual transactions in the electricity market.

With the overall results, we could reject the null hypothesis. Having congestion in the network creates differences in optimal bidding strategy of the financial trader and its welfare impact on market participants, and having optimal virtual bidding in the congested network creates heterogeneous welfare impacts across the network.

This study contributes to the nexus of studies about the impact of virtual transactions on the market efficiency in the wholesale electricity market, identifying welfare impact by introducing optimal virtual bidding traders employing complicated power network governed by physical laws. Most of the previous literature focused on the efficiency defined by the degree of price convergence brought by having a virtual transaction in the electricity market (Borenstein et al. 2008; Hadsell 2011; Haugom and Ullrich 2012). There are few literature that studying the welfare impact of a virtual transaction (Giraldo et al. 2017) using a stylized parsimonious two-settlement market model based on a single node.

However, it is important to note that the actual complexity of a wholesale power market is far greater than the ones used in this study. For example, the aggregated zonal level ISO-NE test case has 8 zones to bid, the financial traders in PJM market can make transactions at more than 10,000 nodes. Hence, the complicated network with transmission constraints ruled by loop flow would result in more complex bidding behavior, and the direct impact of virtual transactions on market efficiency may be ambiguous. In addition, we only consider the risk-neutral profit-maximizing virtual traders in the network in the study. They may have different intention such as hedging and co-bidding with their underlying physical assets or congestion products such as generation fleets, loads and financial transmission rights. While the majority of the financial bidders in the markets are actually from purely speculative institutions, financial products are often used as part of the physical asset and grid management.

In the future, this line of research can be further developed by using the more complicated network. Securing higher resolution data that more nearly reflects the wholesale electricity network may be very useful to understand the sophisticated bidding strategies possible with complicated network constraints. In addition, besides using the virtual products only for profit-maximizing speculative behavior, market participants may use them differently for their business purposes, such as hedging the real-time price volatility of load-serving entity with different risk preferences. It may reveal different welfare impacts on market participants.

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APPENDIX A. Notation

Indices and sets

b,i,j	Set of buses in the electricity network
b(ae)	Subset of buses having exponential inverse supply function for all generators
b(aq)	Subset of buses having quadratic inverse supply function for all generators
b(pe)	Subset of buses having exponential inverse supply function for peakers
b(pq)	Subset of buses having quadratic inverse supply function for peakers
ij	Set of connected arc lines in the electricity network (i: injection bus on arc line, j: withdrawal bus on arc line)
s	Set of positive mass points in the empirical distribution of real-time demand

Parameters

$d1_b$	Demand in Day-Ahead (DA) market at bus b
$d2_{bs}$	Demand in Real-Time (RT) market at bus b in scenario s
Δ_{bs}	Demand deviation in RT market at bus b in scenario s
Prob(s)	Probability of scenario s
$a1_b$	Supply function parameter of aggregated generators in DA market at bus b
$b1_b$	Supply function slope of aggregated generators in DA market at bus b
$a2_b$	Supply function parameter of aggregated generators in RT market at bus b
$b2_b$	Supply function slope of aggregated generators in RT market at bus b
$PTDF_{ijb}$	Power transfer distribution factor matrix. It defines how much increased power flowing on line ij with extra unit of power injection (withdrawal) at bus b and withdrawal (injection) at reference bus
G_b^{max}, G_b^{min}	Maximum and minimum capacity of aggregated generators located at bus b
F_{ij}^{max}	Maximum capacity on line ij
RP	Reservation price of consumer demand
$LMP1_b$	Locational marginal price in DA market at bus b
$LMP2_{bs}$	Locational marginal price in RT market at bus b in scenario s

Variables

V_b	Cleared virtual bids on bus b
$f1_{ij}$	Power flowing in DA market on line ij
$f2_{ijs}$	Power flowing in RT market on line ij in scenario s
$g1_b$	Power generated in DA market at bus b
$g2_{bs}$	Power generated in DA market at bus b in scenario s
$\lambda 1$	Lagrange multiplier of system balance equation in DA
$\lambda 2_s$	Lagrange multiplier of system balance equation in RT in scenario s
$\lambda 1_b$	Lagrange multiplier of bus balance equation in DA at bus b
$\lambda 2_{bs}$	Lagrange multiplier of bus balance equation in RT at bus b in scenario s
$\pi 1_{ij}^{+}, \pi 1_{ij}^{-}$	Lagrange multiplier of line capacity equation in DA on line ij
$\pi 2_{ijs}^{+}, \pi 2_{ijs}^{-}$	Lagrange multiplier of line capacity equation in RT on line ij in scenario s
μ_b^{+}, μ_b^{-}	Lagrange multiplier of generation limit equation in DA at bus b
$\mu_{bs}^{+}, \mu_{bs}^{-}$	Lagrange multiplier of generation limit equation in DA at bus b in scenario s
$S\_\lambda 1$	Slack variable for $\lambda 1$
$S\_\lambda 2_s$	Slack variable for $\lambda 2_s$
$S\_\lambda 1_b$	Slack variable for $\lambda 1_b$
$S\_\lambda 2_{bs}$	Slack variable for $\lambda 2_{bs}$
$S\_\pi 1_{ij}^{+}$	Slack variable for $\pi 1_{ij}^{+}$
$S\_\pi 1_{ij}^{-}$	Slack variable for $\pi 1_{ij}^{-}$
$S\_\pi 2_{ijs}^{+}$	Slack variable for $\pi 2_{ijs}^{+}$
$S\_\pi 2_{ijs}^{-}$	Slack variable for $\pi 2_{ijs}^{-}$
$S\_\mu 1_b^{+}$	Slack variable for $\mu 1_b^{+}$
$S\_\mu 1_b^{-}$	Slack variable for $\mu 1_b^{-}$
$S\_\mu 2_{bs}^{+}$	Slack variable for $\mu 2_{bs}^{+}$
$S\_\mu 2_{bs}^{-}$	Slack variable for $\mu 2_{bs}^{-}$

APPENDIX B. Sandbox Model

B.1 Formulation of the Virtual Trader's Profit Maximization Problem Subject to Day Ahead and Real Time Market Equilibrium

Upper-level: Virtual trader

$$\max \quad \sum_s \sum_b \text{Prob}(s) \cdot V_b (LMP1_b - LMP2_{bs})$$

$$LMP1_b = \lambda 1 - \sum_i \sum_j PTDF_{ijb} \left(\pi 1_{ij}^{+} - \pi 1_{ij}^{-}\right) \qquad \forall_b$$

$$LMP2_{bs} = \lambda 2_s - \sum_i \sum_j PTDF_{ijb} \left(\pi 2_{ijs}^{+} - \pi 2_{ijs}^{-}\right) \qquad \forall_{bs}$$

s.t

Lower-level: Day-Ahead market

$$-\frac{a1_b}{b1_b} + \frac{g1_b}{b1_b} - \lambda 1 + \pi 1_{ij}^{+} \sum_i \sum_j PTDF_{ijb} - \pi 1_{ij}^{-} \sum_i \sum_j PTDF_{ijb} = 0 \qquad \forall_b$$

$$-\sum_b (g1_b - (d1_b - V_b)) + S\_\lambda 1 = 0$$

$$\sum_b PTDF_{ijb} \, (g1_b - (d1_b - V_b)) - F_{ij}^{max} + S\_\pi 1_{ij}^{+} = 0 \qquad \forall_{ij}$$

$$-\sum_b PTDF_{ijb} \, (g1_b - (d1_b - V_b)) - F_{ij}^{max} + S\_\pi 1_{ij}^{-} = 0 \qquad \forall_{ij}$$

$$0 \leq S\_\lambda 1 \perp \lambda 1 \geq 0$$

$$0 \leq S\_\pi 1_{ij}^{+} \perp \pi 1_{ij}^{+} \geq 0 \qquad \forall_{ij}$$

$$0 \leq S\_\pi 1_{ij}^{-} \perp \pi 1_{ij}^{-} \geq 0 \qquad \forall_{ij}$$

Lower-level: Real-Time market

$$\max\left(-\frac{a1_b}{b1_b} + \frac{g2_{bs}}{b1_b}, -\frac{a2_b}{b2_b} + \frac{g2_{bs}}{b2_b}\right)$$

$$-\lambda 2_s + \pi 2_{ijs}^{+} \sum_i \sum_j PTDF_{ijb} - \pi_{ijs}^{-} \sum_i \sum_j PTDF_{ijb} = 0 \qquad \forall_{bs}$$

$$-\sum_b (g2_{bs} - d2_{bs}) + S\_\lambda 2_s = 0 \qquad \forall_s$$

$$\sum_b PTDF_{ijb} \, (g2_{bs} - d2_{bs}) - F_{ij}^{max} + S\_\pi 2_{ijs}^{+} = 0 \qquad \forall_{ijs}$$

$$-\sum_b \mathrm{PTDF}_{ijb}\,(g2_{bs} - d2_{bs}) - F_{ij}^{max} + S\_\pi2^{-}_{ijs} = 0 \qquad \forall_{ijs}$$

$$0 \leq S\_\lambda2_s \perp \lambda2_s \geq 0 \qquad \forall_s$$

$$0 \leq S\_\pi2^{+}_{ijs} \perp \pi2^{+}_{ijs} \geq 0 \qquad \forall_{ijs}$$

$$0 \leq S\_\pi2^{-}_{ijs} \perp \pi2^{-}_{ijs} \geq 0 \qquad \forall_{ijs}$$

APPENDIX C. ISO-NE Test Case Model

C.1 Bilevel Formulation of the Virtual Trader's Profit Maximization Problem Subject to Day Ahead and Real Time Market Equilibrium

Upper-level: Virtual trader

$$\max \quad \sum_s \sum_b \text{Prob}(s) \cdot V_b(\text{LMP1}_b - \text{LMP2}_{bs})$$

$$\text{LMP1}_b = \lambda 1 - \sum_i \sum_j \text{PTDF}_{ijb} \left(\pi 1^+_{ij} - \pi 1^-_{ij}\right) \qquad \forall_b$$

$$\text{LMP2}_{bs} = \lambda 2_s - \sum_i \sum_j \text{PTDF}_{ijb} \left(\pi 2^+_{ijs} - \pi 2^-_{ijs}\right) \qquad \forall_{bs}$$

s.t

Lower-level: Day-Ahead market

$$\max \sum_b \text{RP}_b \cdot d1_b - \sum_{b(aq)} \int_0^{g1_{b(aq)}} a1_{b(aq)} \cdot g + b1_{b(aq)} \cdot g^2 \, dg - \sum_{b(ae)} \int_0^{g1_{b(ae)}} a1_{b(ae)}\left(e^{g \cdot b1_{b(ae)}} - 1\right) dg$$

s.t

$$\sum_b (g1_b - (d1_b - V_b)) \geq 0 \qquad \lambda 1 \geq 0$$

$$\sum_b \text{PTDF}_{ijb} \, (g1_b - (d1_b - V_b)) \leq F_{ij}^{max} \qquad \pi 1^+_{ij} \geq 0 \qquad \forall_{ij}$$

$$-\sum_b \text{PTDF}_{ijb} \, (g1_b - (d1_b - V_b)) \leq F_{ij}^{max} \qquad \pi 1^-_{ij} \geq 0 \qquad \forall_{ij}$$

$$g1_b \leq G_b^{max} \qquad \mu 1^+_b \geq 0 \qquad \forall_b$$

$$-g1_b \leq -G_b^{min} \qquad \mu 1^-_b \geq 0 \qquad \forall_b$$

Lower-level: Real-Time market

$$\max \sum_s \sum_b \text{Prob}(s) \cdot \text{RP}_b \cdot d1_b$$

$$- \sum_s \text{Prob}(s) \cdot \Big(+ \sum_{b(aq)} \int_0^{\min(g1_{b(aq)}, g2_{b(aq)s})} a1_{b(aq)} \cdot g + b1_{b(aq)} \cdot g^2 \, dg$$

$$+ \sum_{b(ae)} \int_0^{\min(g1_{b(ae)}, g2_{b(ae)s})} a1_{b(ae)}\left(e^{g \cdot b1_{b(ae)}} - 1\right) dg$$

$$+ \sum_{b(pq)} \int_0^{\max(g2_{b(pq)s} - g1_{b(pq)}, 0) + \max(g1_{b(pq)} - \text{Base}_{b(pq)}, 0)} a2_{b(aq)} \cdot g + b2_{b(aq)} \cdot g^2 \, dg$$

$$- \sum_{b(pq)} \int_0^{\max(g1_{b(pq)} - \text{Base}_{b(pq)}, 0)} a2_{b(ae)}\left(e^{g \cdot b2_{b(ae)}} - 1\right) dg$$

$$+ \sum_{b(pe)} \int_0^{\max(g2_{b(pe)s} - g1_{b(pe)}, 0) + \max(g1_{b(pe)} - \text{Base}_{b(pe)}, 0)} a2_{b(ae)}\left(e^{g \cdot b2_{b(ae)}} - 1\right) dg$$

$$-\sum_{b(pe)}\int_0^{\max(g1_{b(pe)}-Base_{b(pe)},0)} a2_{b(ae)}\left(e^{g\cdot b2_{b(ae)}}-1\right)dg$$

s.t

$$\sum_b(g2_{bs}-d2_{bs}) \geq 0 \qquad \lambda2_s \geq 0 \qquad \forall_s$$

$$\sum_b PTDF_{ijb}\,(g2_{bs}-d2_{bs}) \leq F_{ij}^{max} \qquad \pi2_{ijs}^{+} \geq 0 \qquad \forall_{ijs}$$

$$-\sum_b PTDF_{ijb}\,(g2_{bs}-d2_{bs}) \leq F_{ij}^{max} \qquad \pi2_{ijs}^{-} \geq 0 \qquad \forall_{ijs}$$

$$g2_{bs} \leq G_b^{max} \qquad \mu2_{bs}^{+} \geq 0 \qquad \forall_{bs}$$

$$-g2_{bs} \leq -G_b^{min} \qquad \mu2_{bs}^{-} \geq 0 \qquad \forall_{bs}$$

C.2 KKT Reformulation of the Virtual Trader's Profit Maximization Problem Subject to Day Ahead and Real Time Market Equilibrium

Upper-level: Virtual trader

$$\max \quad \sum_s \sum_b \mathrm{Prob}(s) \cdot V_b (LMP1_b - LMP2_{bs})$$

$$LMP1_b = \lambda 1 - \sum_i \sum_j PTDF_{ijb} \left(\pi 1^+_{ij} - \pi 1^-_{ij}\right) \qquad \forall_b$$

$$LMP2_{bs} = \lambda 2_s - \sum_i \sum_j PTDF_{ijb} \left(\pi 2^+_{ijs} - \pi 2^-_{ijs}\right) \qquad \forall_{bs}$$

s.t

Lower-level: Day-Ahead market

$$a1_{b(aq)} \cdot g1_b + \beta 1_{b(aq)} \cdot {g1_{b(aq)}}^2 + a1_{b(ae)}\left(e^{g1_{b(ae)} \cdot b1_{b(ae)}} - 1\right)$$

$$-\lambda 1 + \pi 1^+_{ij} \sum_i \sum_j PTDF_{ijb} - \pi 1^-_{ij} \sum_i \sum_j PTDF_{ijb} + \mu 1^+_b - \mu 1^-_b = 0 \qquad \forall_b$$

$$-\sum_b (g1_b - (d1_b - V_b)) + S\_\lambda 1 = 0 \qquad \forall_b$$

$$\sum_b PTDF_{ijb} (g1_b - (d1_b - V_b)) - F_{ij}^{max} + S\_\pi 1^+_{ij} = 0 \qquad \forall_{ij}$$

$$-\sum_b PTDF_{ijb} (g1_b - (d1_b - V_b)) - F_{ij}^{max} + S\_\pi 1^-_{ij} = 0 \qquad \forall_{ij}$$

$$g1_b - G_b^{max} + S\_\mu 1^+_b = 0 \qquad \forall_b$$

$$-g1_b + G_b^{min} + S\_\mu 1^-_b = 0 \qquad \forall_b$$

$$0 \le S\_\lambda 1 \perp \lambda 1 \ge 0$$

$$0 \le S\_\pi 1^+_{ij} \perp \pi 1^+_{ij} \ge 0 \qquad \forall_{ij}$$

$$0 \le S\_\pi 1^-_{ij} \perp \pi 1^-_{ij} \ge 0 \qquad \forall_{ij}$$

$$0 \le S\_\mu 1^+_b \perp \mu 1^+_b \ge 0 \qquad \forall_b$$

$$0 \le S\_\mu 1^-_b \perp \mu 1^-_b \ge 0 \qquad \forall_b$$

Lower-level: Real-Time market

$$(a1_{b(aq)} \cdot \min(g1_{b(aq)}, g2_{b(aq)s}) + b1_{b(aq)} \cdot \min\left(g1_{b(aq)}, g2_{b(aq)s}\right)^2)$$

$$+a1_{b(ae)}\left(e^{\min(g1_{b(ae)},g2_{b(ae)s})\cdot b1_{b(ae)}}-1\right)$$

$$+(a2_{b(pq)}\cdot(\max(g2_{b(pq)s}-g1_{b(pq)},0)+\max(g1_{b(pq)}-Base_{b(pq)},0))+\beta2_{b(pq)}\cdot$$

$$(\max(g2_{b(pq)s}-g1_{b(pq)},0)+\max(g1_{b(pq)}-Base_{b(pq)},0))^2)$$

$$-(a2_{b(pq)}\cdot\max(g1_{b(pq)}-Base_{b(pq)},0)+\beta2_{b(pq)}\cdot\max(g1_{b(pq)}-Base_{b(pq)},0)^2)$$

$$+a2_{b(pe)}\left(e^{(\max(g2_{b(pq)s}-g1_{b(pq)},0)+\max(g1_{b(pq)}-Base_{b(pq)},0))\cdot b2_{b(pe)}}-1\right)$$

$$-a2_{b(pe)}\left(e^{\max(g1_{b(pq)}-Base_{b(pq)},0)\cdot\beta2_{b(pe)}}-1\right)$$

$$-\lambda2_s+\pi2^+_{ijs}\sum_i\sum_j PTDF_{ijb}-\pi^-_{ijs}\sum_i\sum_j PTDF_{ijb}+\mu1^+_{bs}-\mu1^-_{bs}=0 \qquad \forall_{bs}$$

$$-\sum_b(g2_{bs}-d2_{bs})+S\_\lambda2_s=0 \qquad \forall_s$$

$$\sum_b PTDF_{ijb}\,(g2_{bs}-d2_{bs})-F_{ij}^{max}+S\_\pi2^+_{ijs}=0 \qquad \forall_{ijs}$$

$$-\sum_b PTDF_{ijb}\,(g2_{bs}-d2_{bs})-F_{ij}^{max}+S\_\pi2^-_{ijs}=0 \qquad \forall_{ijs}$$

$$g1_{bs}-G_{bs}^{max}+S\_\mu1^+_{bs}=0 \qquad \forall_{bs}$$

$$-g1_{bs}+G_{bs}^{min}+S\_\mu1^-_{bs}=0 \qquad \forall_{bs}$$

$$0\le S\_\lambda2_s\perp\lambda2_s\ge0 \qquad \forall_s$$

$$0\le S\_\pi2^+_{ijs}\perp\pi2^+_{ijs}\ge0 \qquad \forall_{ijs}$$

$$0\le S\_\pi2^-_{ijs}\perp\pi2^-_{ijs}\ge0 \qquad \forall_{ijs}$$

$$0\le S\_\mu1^+_{bs}\perp\mu1^+_{bs}\ge0 \qquad \forall_{bs}$$

$$0\le S\_\mu1^-_{bs}\perp\mu1^-_{bs}\ge0 \qquad \forall_{bs}$$

C.3 Pasting RT inverse supply function

Let

g_DA = DA generation

g_RT = RT generation

P_RT(g_RT) = the pasted inverse supply function

P_DA(g) = the DA inverse supply function (P_DA(0) = 0)

P_P(g) = the peaker only inverse supply function (P_P(0) = 0)

P_RT(g_RT) = P_DA[min(g_RT,g_DA)]
+ P_P[max(g_RT-g_DA,0) + max(g_DA-C_B,0)]
- P_P[max(g_DA-C_B,0)]

If g_RT < g_DA then P_RT = P_DA.

If g_RT > g_DA < C_B then P_RT = P_DA(g_DA) + P_P[g_RT-g_DA].

If g_RT > g_DA > C_B then P_RT = P_DA(g_DA) + P_P[g_RT-C_B] - P_P[[g_DA-C_B].

Note that as g_RT approaches g_DA, P_RT is continuous in every case.

APPENDIX D. ISO-NE Test Case Data

D.1 Generator Quadratic Supply Functions (Krishnamurthy, Li, and Tesfatsion, 2016)

Generator	Zone	Capacity	Parameter_a	Parameter_b	Fuel
Generator_1	NH	1,244	10	0.00020	NUC
Generator_2	CT	1,235	10	0.00023	NUC
Generator_3	CT	881	5	0.00015	NUC
Generator_4	SEMASS	685	7	0.00021	NUC
Generator_5	VT	620	11	0.00022	NUC
Generator_6	RI	612	18	0.00012	BIT
Generator_7	CT	372	20	0.00167	SUB
Generator_8	CT	372	20	0.00167	SUB
Generator_9	RI	244	18	0.00012	BIT
Generator_10	RI	244	18	0.00012	BIT
Generator_11	ME	150	18	0.00012	BIT
Generator_12	WCMASS	144	18	0.00012	BIT
Generator_13	ME	82	18	0.00012	BIT
Generator_14	ME	80	18	0.00012	BIT
Generator_15	NH	48	20	0.00167	BIT
Generator_16	NH	48	20	0.00167	BIT
Generator_17	ME	600	161	0.02367	RFO
Generator_18	SEMASS	559	153	0.03460	RFO
Generator_19	SEMASS	553	192	0.03463	RFO
Generator_20	CT	448	160	0.02360	RFO
Generator_21	RI	435	192	0.03450	RFO
Generator_22	ME	431	233	0.03120	RFO
Generator_23	CT	407	233	0.00598	RFO
Generator_24	NH	400	55	0.00140	RFO

Table continued

Generator_25	CT	400	200	0.02184	RFO
Generator_26	CT	236	192	0.03463	RFO
Generator_27	CT	168	151	0.03470	RFO
Generator_28	CT	131	152	0.03450	RFO
Generator_29	CT	117	192	0.03463	RFO
Generator_30	ME	116	154	0.03460	RFO
Generator_31	CT	81	54	0.00200	RFO
Generator_32	ME	694	23	0.00294	NGLN
Generator_33	ME	685	25	0.00292	NGLN
Generator_34	SEMASS	676	57	0.00292	NGA4
Generator_35	ME	555	90	0.00292	NGA1
Generator_36	RI	516	28	0.00292	NGTN
Generator_37	NH	508	23	0.00292	NGMN
Generator_38	ME	490	23	0.00292	NGMN
Generator_39	CT	448	50	0.00292	NGIR
Generator_40	RI	271	28	0.02290	NGTN
Generator_41	RI	265	57	0.00300	NGA4
Generator_42	RI	249	22	0.00002	NGA4
Generator_43	RI	248	22	0.00002	NGA4
Generator_44	RI	248	22	0.00002	NGA4
Generator_45	ME	245	52	0.00300	NGPN
Generator_46	SEMASS	245	58	0.00400	NGA4
Generator_47	RI	239	58	0.00350	NGA4
Generator_48	WCMASS	238	25	0.00375	NGT2
Generator_49	RI	236	57	0.00372	NGA4
Generator_50	ME	154	90	0.00410	NGA1
Generator_51	RI	149	57	0.00800	NGT4
Generator_52	RI	149	57	0.01300	NGA4

Table continued

Generator_53	RI	149	57	0.01750	NGA4
Generator_54	RI	149	57	0.02200	NGA4
Generator_55	SEMASS	141	57	0.01500	NGA4
Generator_56	WCMASS	141	95	0.00350	NGT1
Generator_57	SEMASS	105	58	0.00500	NGA4
Generator_58	WCMASS	104	100	0.00750	NGT2
Generator_59	CT	44	57	0.02133	NGA4
Generator_60	CT	44	57	0.02133	NGA4
Generator_61	CT	44	57	0.02133	NGA4
Generator_62	CT	43	57	0.02133	NGA4
Generator_63	WCMASS	275	375	0.00420	NGTN
Generator_64	RI	300	400	0.00350	NGTN
Generator_65	RI	325	350	0.00360	NGTN
Generator_66	CT	225	325	0.03420	RFO
Generator_67	WCMASS	150	375	0.00800	NGT2
Generator_68	WCMASS	175	350	0.00900	NGT2
Generator_69	ME	175	300	0.00900	NGPN
Generator_70	ME	85	290	0.00900	NGPN
Generator_71	ME	55	310	0.00900	NGPN
Generator_72	ME	150	325	0.00900	NGTN
Generator_73	ME	200	350	0.00800	NGTN
Generator_74	ME	150	340	0.00750	NGTN
Generator_75	ME	100	375	0.00850	NGTN
Generator_76	ME	125	310	0.00950	NGTN

Note: The quadratic function with generation g (MW) is parameter_a + parameter_b*g^2. NUC: Nuclear, BIT: Coal, NGIR, NGA1, NGA4, NGLN, NGMN, NGPN, NGT1, NGT2, NGT4, NGTN: Natural Gas, RFO: Fuel Oil